Confined Aquifer Steady Flow Calculations

**FIGURE 4.16** Steady flow through a confined aquifer of uniform thickness.

The quantity of flow per unit width, $q'$, may be determined from Darcy’s law:

$$q' = k \frac{S}{h}$$

where $q'$ is the flow per unit width ($\text{L}^3/\text{T} \times \text{A}^2$ or $\text{m}^3/\text{day}$)

$k$ is the hydraulic conductivity ($\text{L}/\text{T}$ or $\text{m}/\text{day}$)

$
\frac{S}{h}$ is the slope of potentiometric surface (dimensionless)

One may wish to know the head, $h_1$, $h_2$, or $h_3$, at some intermediate distance $x$ (L) from $h_1$ and $h_2$. This may be found from the equation:

$$h = h_1 - \frac{x}{h^2}$$

where $x$ is the distance from $h_1$.

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Unconfined Aquifer Steady Flow Calculations

This problem was solved by Dupuit (1863), and his assumptions are known as the Dupuit assumptions. The assumptions are that (1) the hydraulic gradient is equal to the slope of the water table and (2) for small water-table gradients, the mountains are flat, and the equipotential lines are vertical. Solutions based on these assumptions have proved to be useful in many practical problems. However, the Dupuit assumptions do not allow for a seepage face above the outflow side.

From Darcy’s law,

\[
\varepsilon = -\frac{\text{d}h}{\text{d}x}
\]

where \(h\) is the saturated thickness of the aquifer, \(x = L, B, x = L, h = b_2\).

Equation 4.75 may be set up for integration with the boundary conditions:

\[
\int_{x_1}^{x_2} \left( \frac{\partial h}{\partial x} \right) dx = -Q
\]

Integration of the preceding yields

\[
\frac{h}{2}\left( \frac{L}{b} \right) = \frac{h_2}{2} + \frac{Q}{2}
\]

Substitution of the boundary conditions for \(x = 1\) and \(x = 2\) yields

\[
\varepsilon = \frac{Q}{L}
\]
For steady flow, any change in flow through the prism must be equal to a gain or loss of water across the water table. This could be infiltration or evapotranspiration. The net addition or loss is at a rate of \( u \), and the volume change within the initial volume is \( u \, dt \, dy \). Hence, the area of the surface. If it represents evapotranspiration, it will have a negative value. As the change in flow is equal to the new addition,

\[
x \frac{d}{dx} \left( \frac{dh}{dy} dy \right) dx - y \frac{d}{dy} \left( \frac{dh}{dx} dx \right) dy = u \, dt \, dy
\]

We can simplify Equation 4.64 by dropping out \( dt \, dy \) and combining the differentials:

\[
-x \frac{d}{dx} \left( \frac{dh}{dy} \right) + y \frac{d}{dy} \left( \frac{dh}{dx} \right) = 2u
\]

If \( u = 0 \), then Equation 4.65 reduces to a form of the Laplace equation:

\[
\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0
\]

If flow is in only one direction and we align the x-axis parallel to the flow, then there is no flow in the y-direction, and Equation 4.65 becomes:

\[
\frac{\partial^2 h}{\partial x^2} = \beta
\]

Integration of this equation yields the expression:

\[
h = \frac{\beta x}{2} + c(x) + c_1
\]

where \( c_1 \) and \( c(x) \) are constants of integration.

The following boundary conditions can be applied: at \( x = 0 \), \( h = h_0 \); at \( x = L \), \( h = h_1 \) (Figure 4.15). By substituting these into Equation 4.66, the constants of integration can be evaluated with the following results:

\[
h = \frac{\beta x}{2} + \left( h_1 - h_0 \right) \frac{x}{L} + h_0
\]

where:
- \( h \) is the head at \( x (L, 0 \text{ or } m) \)
- \( x \) is the distance from the origin (L, 0 or m)
- \( h_0 \) is the head at \( x (L, 0 \text{ or } m) \)
- \( h_1 \) is the head at \( x (L, 0 \text{ or } m) \)
- \( h_0 \) is the head at \( x (L, 0 \text{ or } m) \)
- \( h_1 \) is the head at \( x (L, 0 \text{ or } m) \)
- \( \beta \) is the hydraulic conductivity \( (L/T, 0 \text{ or } m/s) \)
- \( \alpha \) is the exchange rate \( (L^2, 0 \text{ or } m^2/s) \)

Dispersion can be used to find the elevation of the water table anywhere between two points located at the same point. If the saturated thickness of the aquifer is known, the two points can be.
For the case in which there is no infiltration or evaporation, \( v = 0 \) and Equation 4.20 reduces to:

\[
    h = \sqrt{\frac{2g}{k} \left( z - z_0 \right)}
\]

By differentiating Equation 4.20, and because \( \frac{dM}{dt} = -\frac{dh}{dt} \), it may be shown that the flowmeter and velocity \( v \) at any section a distance from the origin is given by:

\[
    v = \frac{K(2g)^{1/2}}{2a \left( \frac{a^2}{3} \right)}
\]

where:
- \( \frac{a}{2} \) is the free surface width normal to the flow
- \( y \) is the distance from the origin \( z_0 \)
- \( k \) is the hydraulic conductivity (ft/day or m/day)
- \( h \) is the head at the origin \( z_0 \)
- \( K \) is the Lacey head at the end of the channel
- \( h_0 \) is the head of the water table at the origin

Once the distance from the origin to the water table has been found, the elevation of the water table at the discharge can be obtained by substituting \( x = 0 \) in Equation 4.20.

\[
    h_{w_0} = \frac{K \sqrt{2g}}{2} \left( \frac{a}{3} \right) \left( \frac{a^2}{3} \right)
\]

Where, \( h_{w_0} \) is the maximum elevation, which is the elevation of the water table divide.

An unconfined aquifer has a hydraulic conductivity of 0.002 ft/day and an effective porosity of 0.20. The aquifer is a layer of sand with a uniform thickness of 30 ft, as measured from the land surface. At well 1, the water table is 31 m below the land surface. At well 2, located some 175 m away, the water table is 20 m below the land surface. What are (a) the discharge per unit width \( Q \), the average linear velocity \( v \) at well 1, and (c) the water table elevation midway between the two wells?

Part A: From Equation 4.12:

\[
    K = \frac{a^2}{2} \left( \frac{a^2}{3} \right)
\]

\[
    h_0 = \frac{K \sqrt{2g}}{2} \left( \frac{a^2}{3} \right)
\]

\[
    \frac{a}{2} = \frac{31}{30} \quad \frac{a}{3} = \frac{20}{30} \quad a^2 = 30 \times 30
\]

\[
    k = \frac{30^2}{2} \times \frac{30^2}{3} = 1800 \text{ ft/day}
\]

\[
    \frac{\Delta h}{\Delta t} = \frac{2g}{k} \left( \frac{a^2}{2} \right)
\]

\[
    v = \frac{\Delta h}{\Delta t} \frac{a^2}{2} \left( \frac{a^2}{3} \right)
\]

\[
    v = \frac{2g}{k} \left( \frac{a^2}{2} \right) \left( \frac{a^2}{3} \right)
\]

Part B: From Equation 4.13:

\[
    h_0 = \frac{K \sqrt{2g}}{2} \left( \frac{a^2}{3} \right)
\]

\[
    h_0 = \frac{1800 \times 2}{2} \times \frac{30^2}{3} = 270 \text{ ft}
\]

\[
    h_0 = \frac{1800 \times 2}{2} \times \frac{30^2}{3} = 270 \text{ ft}
\]
A small weir constructed across a canal 1080 m away. The weir has a semicircular notch of 2.43 m radius. The water surface is at elevation of 23.0 M. The water surface is at an elevation of 21.8 M. (a) The maximum water surface elevation, (b) the daily discharge per 100 ft is the same, and (c) the daily discharge per 100 ft is the same.

Part A: From Equation 47B:

\[ d = \frac{I}{\pi R} \]

\[ A_2 = 2 \pi R \]

\[ I = 23.0 \text{ M} \]

\[ A_2 = 39 \text{ ft}^2 \]

\[ w = 10.8 \text{ ft/day} \text{ due to } 1.8 \text{ ft/day} \text{ evaporation} \]

\[ 0.0001 \text{ ft/day} \]

\[ d = \frac{I}{\pi R} \]

\[ 100 \text{ ft from the weir} \]

Part B: From Equation 47C:

\[ h_m = \frac{I}{\pi R} \sqrt{\frac{R}{2}} \ L \]

\[ = \frac{23.0}{\pi 2.43} \sqrt{\frac{2.43}{2}} \ L \]

\[ = 20 \text{ ft} \]

Part C: From Equation 47D, 47E: (a) \( x = 0 \)

\[ \frac{23.0 - h_m}{2} - \frac{23.0 - h_m}{2} \]

\[ = 10.5 \text{ ft} \]

The negative sign indicates that flow is in the opposite direction from into the canal.