Content

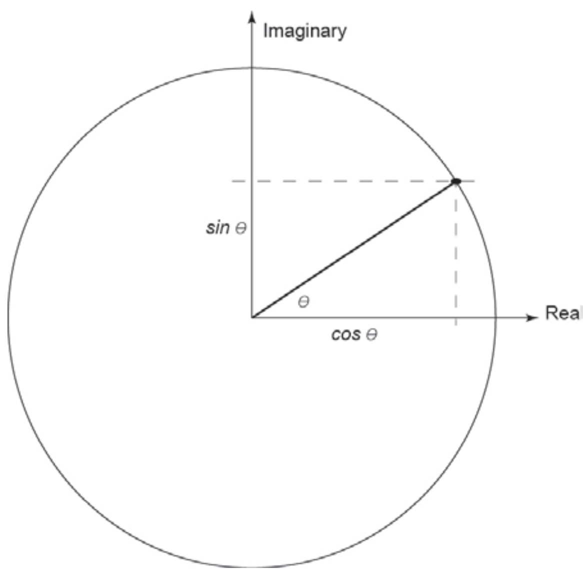
COMPLEX NUMBERS

ARGAND DIAGRAM

Leonhard Euler's (1707-1783 – wikipedia) "magical" formula.

$$e^{i\theta} = \cos\theta + i\sin\theta \quad (1.1)$$

I recommend you explore the Mathematica Demonstration by Jim Kaiser.



When

$\theta = 0$

$$e^{i0} = \cos 0 + i\sin 0$$

$$e^{i0} = 1$$

When

$\theta = \pi$

$$e^{i\pi} = \cos\pi + i\sin\pi$$
$$e^{i\pi} = -1$$

If we consider the following cases we can derive some well-known identities:

Let

$e^{-i\theta} = \cos\theta - i\sin\theta$ (1.2)

If we add (1.3) and (1.4), then

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$ and

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

When, applied to the following representation of a harmonic plane wave:

$\vec{u} = A_0 \hat{u} e^{ik(\vec{n}.\vec{x} - Vt)}$ gives us:

$$\vec{u} = A_0 u\left[\cos\left[k(\vec{n}\bullet\vec{x} - Vt_0\right] + i\sin\left[k(\vec{n}\bullet\vec{x} - Vt_0\right]\right]$$

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From Boas, p. 58 ff

Some of the differential wave equations you will see will require the use of complex numbers. In particular when we examine the solutions to the wave equation when body waves meet the surface of the earth, a "free surface".

A general complex number is written as

$$c = a + ib$$

Note that i IS NOT electrical current but the square root of -1.

In the above expression a is real and b is imaginary (although it is just a real number)

We can see an example of its use in the solution to the simple quadratic equation:

$$x^2 + 2x + 2 = 0$$

The solution can be written as:

$$\begin{aligned} x &= \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1} \\ x &= \frac{-3 \pm \sqrt{-4}}{2} \\ &= \frac{-3 \pm 2i}{2} \\ &= \frac{-3}{2} \pm i \end{aligned}$$

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