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## HARMONIC PLANE WAVE

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#### INTRODUCTION

One of the most common ways of representing waves is to consider the representation of a **harmonic plane wave**. A harmonic plane wave makes a few simplifying assumptions about waves.

- **First**, that we are far enough away from the source that we **do not sense the curvature** of the wavefront.
- **Second**, that we can decompose the wave into individual sinusoidal waves each with different wavelengths or frequencies.
- **Third**, that the medium is **isotropic** so that the rays are normal to the wavefront.

So now, we represent the wave as follows:

$$\vec{u} = A_0 \hat{u} e^{ik(\vec{n} \cdot \vec{x} - Vt)}$$

Where  $\vec{u} = (u_1, u_2, u_3)$  indicates the direction of particle motion at this given location.

$\hat{u}$  is the unit particle displacement vector,  $A_0$  is the value of the displacement,

$\vec{n} = (n_1, n_2, n_3)$  is the propagation vector and indicates the direction of travel of the wave at a location in space,  $\vec{x} = (x_1, x_2, x_3)$  and  $\vec{n} \cdot \vec{x}$  is the equation of the wave front.

If the position vector and propagation vector are parallel then the dot product equals the distance between the origin and the location of the wave front. Multiplying by  $k$  and subtracting the second term will reveal where within the wavelength you are located.

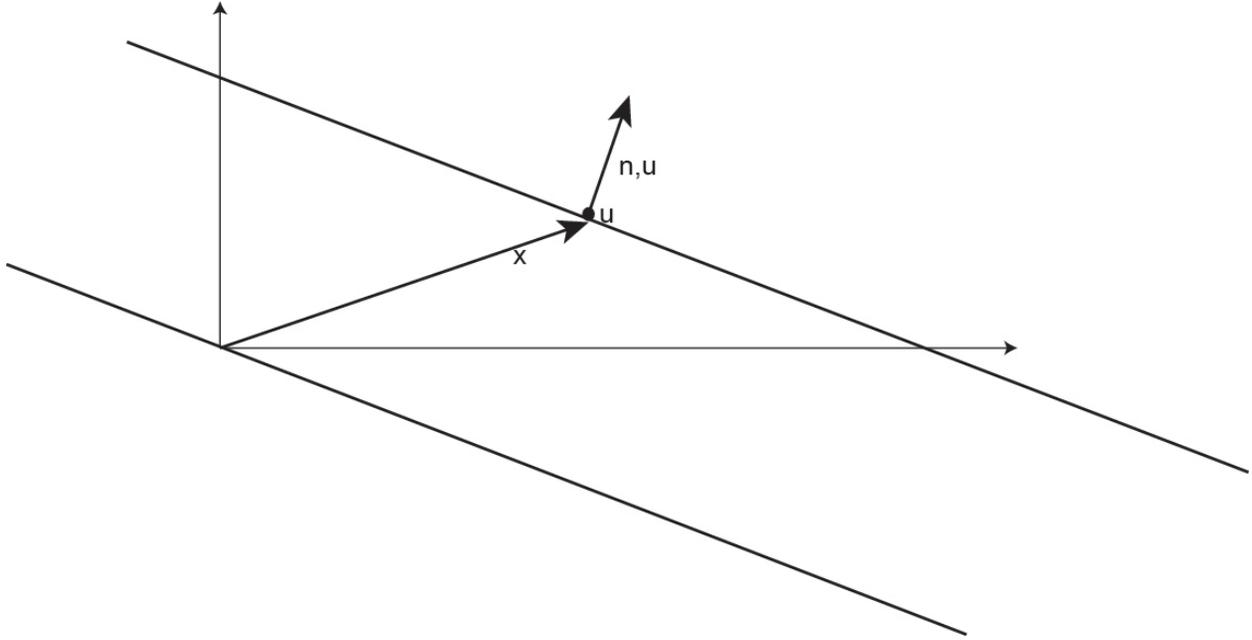
We call this a harmonic solution because it is periodic (recall Euler's equation). The real part of the displacement is given by the cosine portion of Euler's equation.

$\kappa = \frac{2\pi}{\lambda}$  is angular the wavenumber,  $t$  is time since the wave left the source, and  $V$  is the speed of the wave at that point through space.

Note also that  $\lambda$  is the wavelength and  $\omega (= 2\pi f)$  is the angular frequency

Leonhard Euler's formula:

$$e^{i\theta} = \cos \theta + i \sin \theta$$



See accompanying *Mathematica* notebook.