ELASTIC MODULI and PHYSICAL PROPERTIES of ROCKS

Content

FLUIDS

YOUNG'S MODULUS (E)

$$E = \frac{\frac{\Delta F}{A}}{\frac{\Delta L}{L_0}}$$

This modulus reflects the stiffness of earth materials. E is the ratio of **stress to strain**. If our aim is to lengthen or shorten a rock without actually breaking it, the greater the value of E, the larger the stress that is needed to achieve the deformation. Strain is non-dimensional and so the units of E are those of stress.

BULK MODULUS OR INCOMPRESSIBILITY (*k*)

(Pa) -> Acoustic Wave Equation

The bulk modulus is another elastic constant that reflects the resistance of the material to an overall gain or loss of volume in conditions of hydrostatic stress (P_h). If the P_h increases then the volume will decrease and the volume change will be negative. If the volume increases, P_h will decrease. P_h always has positive value and the negative sign in the compensation (R.H.S.) keeps this relation valid. A material that has a large bulk modulus could be imagined to consist of very tightly bound spring and ball materials which will quickly transmit a wave. Ideally, elastic deformation is instantaneous but we know that there is some delay in transmitting a deformation. The stiffer the material then the faster a vibration should travel through the material.

$$k = -\frac{\Delta P}{\frac{\Delta V}{V}}$$

Note that

$$\lambda + 2\mu = k + \frac{4}{3}\mu$$

POISSON'S RATIO

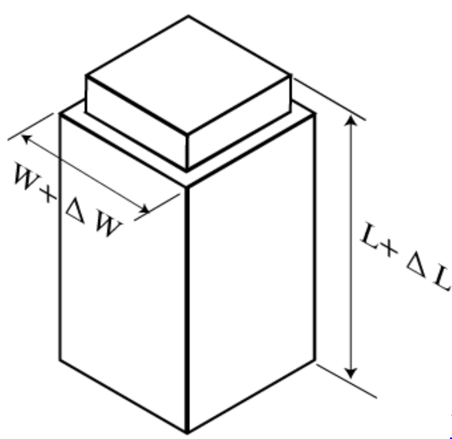
$$\sigma = -\frac{\frac{\Delta W}{W}}{\frac{\Delta L}{L}} \qquad \text{(Pa)}$$

Poisson's ratio (σ) that relates deformation in materials at directions at right angles to each other. Immediately we can predict that perhaps this same value will tell us something of the ratio of compressional to shear waves, since they too deform materials in directions that are at right angles to each other. With σ we can determine the ratio of transverse contraction to longitudinal extension.

Natural materials have Poisson's ratios between 0 and ½. When [illegible] Poisson's ratio equals ¼.

Poisson's ratio is often related to the $\frac{V_P}{V_S}$ ratio (>Mathematica Link)

where, $\frac{V_P}{V_S} = \sqrt{\frac{2(1-\nu)}{(1-\nu)}}$

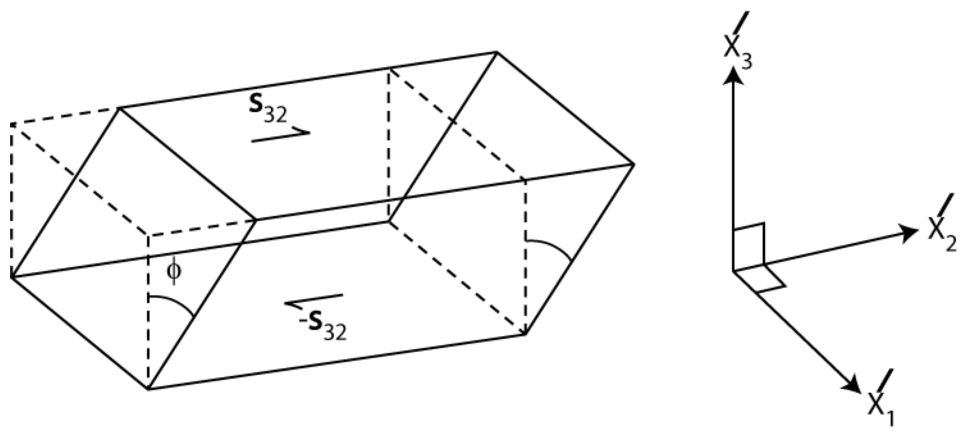


PPTX

SHEAR MODULUS (μ)

$$\mu = \frac{\frac{\Delta F}{A}}{\varphi} \text{ (Pa)}$$

This valuable property tells us ahead of time how stiff a material is to shearing deformation. If a material is very stiff to attempted shearing, then it will transmit the shear energy very quickly. The shear modulus is the ratio the shear stress needed to deform a material by a given angle (measured as the tangent of the deformation angle). As strain has no units, then the shear modulus will have units of Pascals.



Shear modulus is a property that is strain-rate dependent and is not usually a concern unless we are trying to compare shear moduli collected by different techniques. Seismically derived shear moduli are typically greater than those measured by in-lab strength tests because seismic measurements are made under elastic, low-strain conditions. In the case of a simple elastic material, where non-destructive, low-strain is implicitly assumed, the seismic shear modulus (G_{max}) provides an upper-bound measurement of strength and is related to the shear-wave velocity (V_S) by the following relation, where ϱ is the wet bulk density:

$$G_{\max} = V_S^2 \rho^2$$

For an average V_S of 100 m/s and density of 1700 kg/m^3, G_{max} is of order 10^{11} Pa. In comparison, engineering-shear strength measurements carried out at larger strains (Figure 4) are orders of magnitudes lower (Verruijt, p. 391).

The threshold shear strain (at which deformation turns plastic) is a function of the inverse of the shear modulus of the particles and the confining stress raised to the 2/3 power. (Schneider, 1999)

Q. Derive the relation between Poisson's ratio and V_P and V_S.

ELASTIC MODULI IN FLUIDS

When we deal with fluids, we can assume that these materials can not sustain even small shear stresses:

$$\mu \sim 0$$

The P-wave velocity becomes totally dependent on the bulk modulus (*k*), so that:

$$V_P = \sqrt{\frac{k}{\rho}}$$

In a fluid only one of Lamé's parameters is non-zero: λ

What happens to Poisson's ratio when the shear modulus becomes negligible?

Normally Poisson's ratio (*n*) depends on the shear-velocity property as follows:

$$\nu = \frac{\frac{1}{2}\left(\frac{V_P}{V_S}\right)^2 - 1}{\left(\frac{V_P}{V_S}\right)^2 - 1}$$, but when there is no detectable shear wave developed in the purely 'acoustic medium' we have as follows:

As $V_S \to 0$,

$$\nu \to \frac{\frac{1}{2}\left(\frac{V_P}{V_S}\right)^2 - \cancel{1}}{\left(\frac{V_P}{V_S}\right)^2 - \cancel{1}}$$

$$\nu \to \frac{1}{2}$$

and

, because of the relative insignificance of 1's, and that then the Vp/Vs ratios divide into each other to result in 1, thereby leaving the constant ½ as the limit.

NEGATIVE POISSON'S IN AUXETIC FOAMS

See http://www.youtube.com/watch?v=PLDbSWSm5i8

COMMON ELASTIC MODULI IN ROCKS

Often you will read that Poisson's ratio for rocks is about ¼ .

Because the Vp/Vs ratio can be expressed as a function of Poisson's ratio we have the following:

$$\frac{\mathrm{Vp}}{\mathrm{Vs}} = \sqrt{\frac{2(1-\nu)}{(1-2\nu)}}$$ so that

for $\nu = 1/4 \rightarrow \qquad V_P / V_S = \sqrt{3}$

For normal rocks as well, a good Vp-to-Vs ratio to remember is approximately 2. For this case the Poisson's ratio will be about 1/3. That is, when

$$V_P / V_S \sim 2 \ , \ \nu \sim \frac{1}{3}$$

COMMON ELASTIC MODULI IN SOFT MATERIALS

If the shear modulus were truly equal to nothing, then materials would not change their girth in response to a pull or a squeeze and Poisson's ratio would be 1/2. However, even in some materials we think of as being practically fluids, the shear modulus is very small but non-zero. For example, in the tidal flats of Vancouver Koichi Hayashi founds Vs as low as 23 m/s. In medical imaging we use can find that soft human tissue can propagate shear waves as low as 1-10 m/s at frequencies of 10^6Hz. (Fink and Tanter, 2010—also image below). The shear wavelength is about .01m, so the frequency of the shear wave propagation is about 100-1000 Hz.

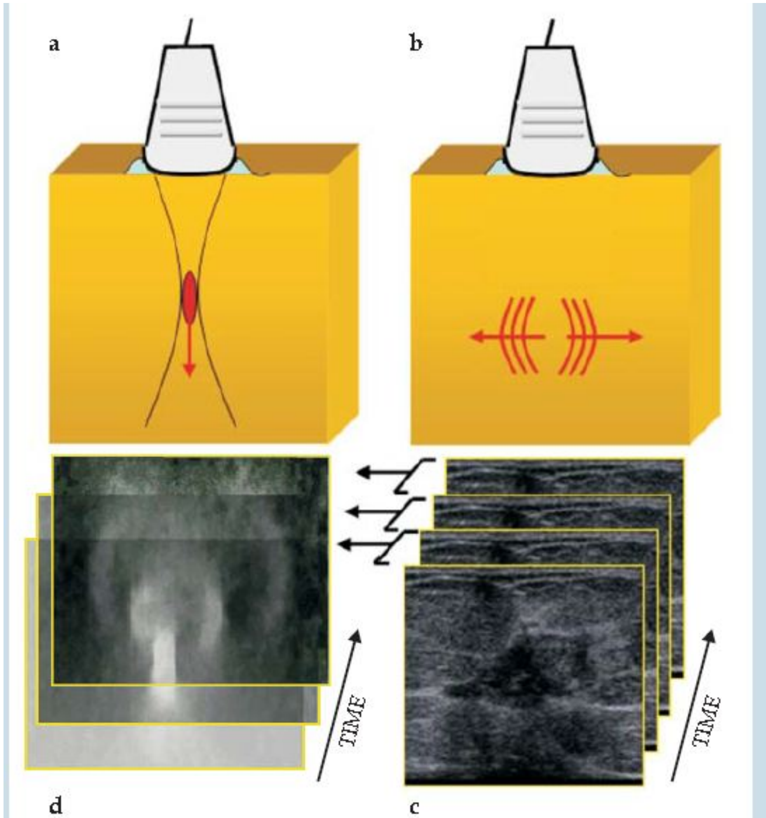


Figure 2. Elasticity imaging. (a) An ultrasonic probe generates a force in the focal area that pushes in the direction of beam propagation. **(b)** Relaxation of the pushed tissue generates a low-frequency shear wave. The ultrasonic array then switches into an ultrafast imaging mode. **(c)** Ultrasonic images, dominated by speckle patterns, are created and then stored into memories. **(d)** Correlation of the speckle patterns in sequential frames yields an image of the tissue displacement induced by the shear wave.

By comparison conventional ultrasound imaging uses acoustic waves that travel at about 1500 m/s for frequencies about 10^6 -10^7 Hz.

What is the wavelength of such an acoustic wave?

Often, predictions of seismic velocities in saturated-unsaturated unconsolidated sediments require an estimate of the effective values of the wet bulk (K_{wet}), wet shear moduli (G_{wet}) and wet bulk density (ρ_{wet}) of the medium where

$$V_P = \sqrt{\frac{K_{wet} + \frac{4}{3} G_{wet}}{\rho_{wet}}} \text{ and } V_S = \sqrt{\frac{G_{wet}}{\rho_{wet}}}.$$

At the low-frequency limit in Gassman (Gassmann, 1951)- Biot theory (Biot, 1956) K_{wet} is relatable to the reference bulk modulus of the framework of mineral grains (K_{ref}) whose porosity is ϕ, the bulk modulus of the minerals comprising the sediment (K_{min}), and the bulk modulus average of the pore fluids (K_{fl}), as follows:

$$\frac{K_{wet}}{K_{\min} - K_{wet}} = \frac{K_{ref}}{K_{\min} - K_{ref}} + \frac{K_{fl}}{\phi\left(K_{\min} - K_{fl}\right)};$$ (Mavko et al., 1998, p. 168), and where the shear modulus (G_{ref}) is unchanged by the pore fluids, which in our case are represented by air and water.

The bulk modulus of the pore space is a weighted harmonic mean of the bulk moduli of the pore constituents (Gassmann, 1951). When the two pore constituents are water and air, the bulk modulus of the pore space fluids (K_{fl}) can be calculated as:

$$\frac{1}{K_{fl}} = \frac{S_w}{K_{water}} + \frac{1 - S_w}{K_{air}}$$

where S_w is water saturation, K_{water} is the bulk modulus of water, and K_{air} is the bulk modulus of air.

In addition, K_{ref} and G_{ref} can be provided by a generalized Hertz-Mindlin (Mindlin, 1949) contact theory extended to a randomly disordered, stack of spheres as:

$$K_{ref} = \sqrt[3]{\frac{C^2(1-\phi)^2 G_{\min}{}^2}{18\pi^2 (1-\upsilon)^2} P_{eff}},$$

$$G_{ref} = \left(\frac{5-4\upsilon}{5(2-\upsilon)}\right) \sqrt[3]{\frac{3C^2(1-\phi)^2 G_{\min}{}^2 P_{eff}}{2\pi^2 (1-\upsilon)^2}}$$ (Mavko et al., 1998)

where C (coordination number) is the average number of contacts between grains,

$G_{\min}$ is the shear modulus,

ν is Poisson's ratio of the mineral grain, and $P_{eff,}$ the effective confining stress between grains.

An increase in saturation in the sand body increases the overall bulk density and through hydrostatic buoyancy, may also decrease the effective confining stress-- $P_{eff} = (\rho_{\min} - \rho_{water})(1-\phi)gz$, (Velea et al., 2000)--both processes act to decrease the overall V_P.

Modifications to the basic assumptions such as the actual smoothness of grains and direct grain contact interaction may limit the accuracy of these velocity predictions (Bachrach et al., 2000; Velea et al., 2000). Intrinsic attenuation may result from friction between the grains due to variations in size and roundness. In a forward modeling approach, these parameters could be iteratively adjusted to best match velocity estimates and used for comparison between different localities.

Total effective stress (Lu and Likos, 2006)

Total effective stress at the grain contacts is used to calculate matrix elasticity in Hertz-Mindlin theory. In the absence of direct measurements, total effective stress can be estimated from the sum of forces acting on the granular matrix:

$$P = (\sigma_T - u_a) + \sigma'_S + \sigma_{CO}$$

where σ_T is the total external stress, u_a is pore-pressure, σ'_S is soil suction stress (Lu and Likos, 2006), and σ_{CO} is apparent tensile stress at the saturated state caused by cohesive or physiochemical forces (Bishop *et al.*, 1960). Physiochemical forces are local forces arising from individual contributions from van der Waals attractions, electrical double layer repulsion, and chemical cementation effects (Lu and Likos, 2006). Saturated cohesion (σ_{CO}) is constant for different soil types and taken from literature Other total effective stress components are accounted for in separate sections of the appendix.