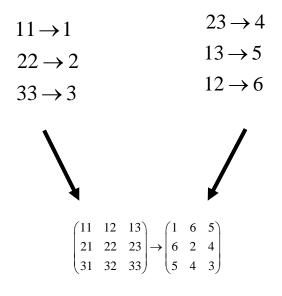
Voigt Notation for Reducing the Dimension of a Tensor

In matrix form, we can use also use the Voigt notation to write out the non-diagonalized isotropic stiffness tensor where the stress and strain can be represented as a six-component vector rather than a nine-element square matrix:

Additional simplification of the stress-strain can be realized through simplifying the matrix notation for stresses and strains. We can replace the indices as follows:



Voigt Notation

Isotropic stress tensor

$$\text{where, } \boldsymbol{\mathcal{E}}_{I} \Leftrightarrow \begin{pmatrix} \boldsymbol{\varepsilon}_{1} \\ \boldsymbol{\sigma}_{2} \\ \boldsymbol{\sigma}_{3} \\ \boldsymbol{\sigma}_{4} \\ \boldsymbol{\sigma}_{5} \\ \boldsymbol{\sigma}_{6} \end{pmatrix}^{=} \begin{pmatrix} 2\mu + \lambda & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & 2\mu + \lambda & 0 & 0 & 0 \\ \lambda & \lambda & 2\mu + \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{pmatrix} \begin{pmatrix} \boldsymbol{\mathcal{E}}_{1} \\ \boldsymbol{\mathcal{E}}_{2} \\ \boldsymbol{\mathcal{E}}_{3} \\ \boldsymbol{\mathcal{E}}_{6} \\ \boldsymbol{\mathcal{E}}_{6} \end{pmatrix} (1.1)$$

$$\text{where, } \boldsymbol{\mathcal{E}}_{I} \Leftrightarrow \begin{pmatrix} \boldsymbol{\mathcal{E}}_{1} \\ \boldsymbol{\mathcal{E}}_{2} \\ \boldsymbol{\mathcal{E}}_{3} \\ \boldsymbol{\mathcal{E}}_{4} \\ \boldsymbol{\mathcal{E}}_{5} \\ \boldsymbol{\mathcal{E}}_{6} \end{pmatrix} \quad \text{and,} \quad \boldsymbol{e}_{ij} = \begin{pmatrix} \boldsymbol{\mathcal{E}}_{1} & \frac{1}{2}\boldsymbol{\mathcal{E}}_{6} & \frac{1}{2}\boldsymbol{\mathcal{E}}_{5} \\ \frac{1}{2}\boldsymbol{\mathcal{E}}_{6} & \boldsymbol{\mathcal{E}}_{2} & \frac{1}{2}\boldsymbol{\mathcal{E}}_{4} \\ \frac{1}{2}\boldsymbol{\mathcal{E}}_{5} & \frac{1}{2}\boldsymbol{\mathcal{E}}_{4} & \boldsymbol{\mathcal{E}}_{3} \end{pmatrix}$$

Recall that:
$$e_{ij} = \begin{pmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{pmatrix}$$

Anisotropic stress tensor

If we assume the anisotropy to be weak in a medium then we can approximate the stiffness matrix as the following. As most sediments consist of clays, the symmetry of clays is a good first approximation to the seismic symmetry seen in nature, or the Transverse Isotropy with a vertical axis of symmetry, or **VTI** (Tsvankin, 2012)

(σ_1)	(c_{11}	$c_{11} - 2c_{66}$	0	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathcal{E}_1 \\ \end{pmatrix}$
σ_2		$c_{11} - 2c_{66}$	c_{13}	0	$0 0 0 \parallel \mathcal{E}_2 \mid$
σ_{3}		c_{13}		c_{33}	$0 0 0 \ \mathcal{E}_3 \ $
σ_4		0	0	0	$c_{55} 0 0 \mid \mathcal{E}_4$
σ_{5}		0	0	0	$0 c_{55} 0 \parallel \mathcal{E}_5$
$\left(\sigma_{_{6}}\right)$		0	0	0	$\begin{array}{ccc} 0 & 0 & c_{66} \end{array} \left(\boldsymbol{\mathcal{E}}_{6} \right) \end{array}$

Voigt notation helps us handle the 2^{nd} order stress and strain tensors even when we have 6 elastic constants (note that one is a linear combination of the other two) as is with the case of anisotropy