
Petroleum Seismology, spring 2010
Homework #2

Purpose: **Review Vector Algebra**

Q. 1 Demonstrate that the following identities are true with an example.
You can use Matlab, or Mathematica, or Excel, etc. or your own example,

$$\mathcal{E}_{ijk} \mathcal{E}_{ijk} = 6$$

$$\mathcal{E}_{ijk} \mathcal{E}_{ijl} = 2\delta_{kl}$$

1)

Proof:

$$\mathcal{E}_{ijk} = \begin{cases} +1 & \text{if } (i, j, k) \text{ is } (1, 2, 3), (2, 3, 1) \text{ or } (3, 1, 2) \\ -1 & \text{if } (i, j, k) \text{ is } (2, 1, 3), (3, 2, 1) \text{ or } (1, 3, 2) \\ 0 & \text{otherwise : } i = j, j = k \text{ or } i = k \end{cases}$$

So in all the 27 possibilities, only the 6 shown above is none zero

So

$$\begin{aligned} \mathcal{E}_{ijk} \mathcal{E}_{ijk} &= 1 \times 1 + 1 \times 1 + 1 \times 1 \\ &\quad + (-1) \times (-1) + (-1) \times (-1) + (-1) \times (-1) \\ &= 6 \end{aligned}$$

2)

Proof:

*In order to have the result none zero,
we will need to have $i \neq j$.*

(1) If $k = l$,

*we need to choose i, j from the rest two numbers
in order to keep the result none zero,
and there are only two ways to choose them.
in this case,*

$$\varepsilon_{ijk} \varepsilon_{ijl} = 1^2 + (-1)^2 = 2$$

(2) If $k \neq l$,

*They took two probabilities out of three,
if we have $k \neq i$ and $k \neq j$, we will have either $l = i$ or $l = j$
if we have $l \neq i$ and $l \neq j$, we will have either $k = i$ or $k = j$
For both the cases above, we will have*

$$\varepsilon_{ijk} \varepsilon_{ijl} = 0$$

$$\Rightarrow \varepsilon_{ijk} \varepsilon_{ijl} = 2\delta_{kl}$$

Q. 2. Rotate the following vector clockwise by 25 degrees about the third axis..
 (1 2 3)
 Calculate the magnitude of the vector.
 Use Excel, Matlab or Mathematica to carry out the rotation.
 Hand in your code and results.

Here are the Matlab codes:

```
*****
*           hw2.m
*****
%Purpose: Caculate the magnitude of the vector and rotate it along the z
%      axis
%Author:Guofeng Yuan
%Date:02/04/2010

%the vector
clear;
clc;
a=[0,0,0];
b=[1,2,3];
%The magnitude of the vector
mag1=norm(b);
%Draw the first vector
vectarrow(a,b,'red');
hold on;
%Rotation angle
theta=25*pi/180;
%The rotation matrix
A=[cos(theta),sin(theta),0;-sin(theta),cos(theta),0;0,0,1];
c=A*b';
%The vector after rotation
d=c';
mag2=norm(d);
%Draw the second vector
vectarrow(a,d,'blue');
%Add title and legend
leg=legend('Before Rotation','After Rotation');
h=title({'Vector (1,2,3) Rotated Clockwise by 25 degrees about z axis '; ['The Magnitude of this
Vector is ',num2str(mag1), "]});
set(h,'fontsize',18);
set(leg,'fontsize',20);
hold off;

set(gcf,'PaperUnits','inches','PaperPosition',[0 0 12 12])
print -djpeg 'myfig.jpg' -r100
*****
```

Here I used a function to draw the vector which I got from a example from mathworks website, and I modified it to fit my needs.

<http://www.mathworks.com/matlabcentral/fileexchange/7470-plot-2d3d-vector-with-arrow>

```
*****
*                               vectarrow.m
*****
function vectarrow(p0,p1,clr)
%Purpose: Arrowline 3-D vector plot.
% Author: Rentian Xiong 04/18/2005
% Modified by:Guofeng Yuan 02/04/2010

if max(size(p0))==3
    if max(size(p1))==3
        x0 = p0(1);
        y0 = p0(2);
        z0 = p0(3);
        x1 = p1(1);
        y1 = p1(2);
        z1 = p1(3);
        colorplot=plot3([x0;x1],[y0;y1],[z0;z1]); % Draw a line between p0 and p1
        set(colorplot,'Color',clr);
        p = p1-p0;
        alpha = 0.1; % Size of arrow head relative to the length of the vector
        beta = 0.1; % Width of the base of the arrow head relative to the length

        hu = [x1-alpha*(p(1)+beta*(p(2)+eps)); x1; x1-alpha*(p(1)-beta*(p(2)+eps))];
        hv = [y1-alpha*(p(2)-beta*(p(1)+eps)); y1; y1-alpha*(p(2)+beta*(p(1)+eps))];
        hw = [z1-alpha*p(3);z1;z1-alpha*p(3)];
        hold on
        arrowplot=plot3(hu(:),hv(:),hw(:),'HandleVisibility','off') % Plot arrow head
        grid on
        xlabel('x')
        ylabel('y')
        zlabel('z')
        set(arrowplot,'Color',clr);
        hold off;

    else
        error('p0 and p1 must have the same dimension')
    end
else
    error('this function only accepts 3D vector')
end
*****
```

Here is the output image

Vector (1,2,3) Rotated Clockwise by 25 degrees about z axis
The Magnitude of this Vector is 3.7417

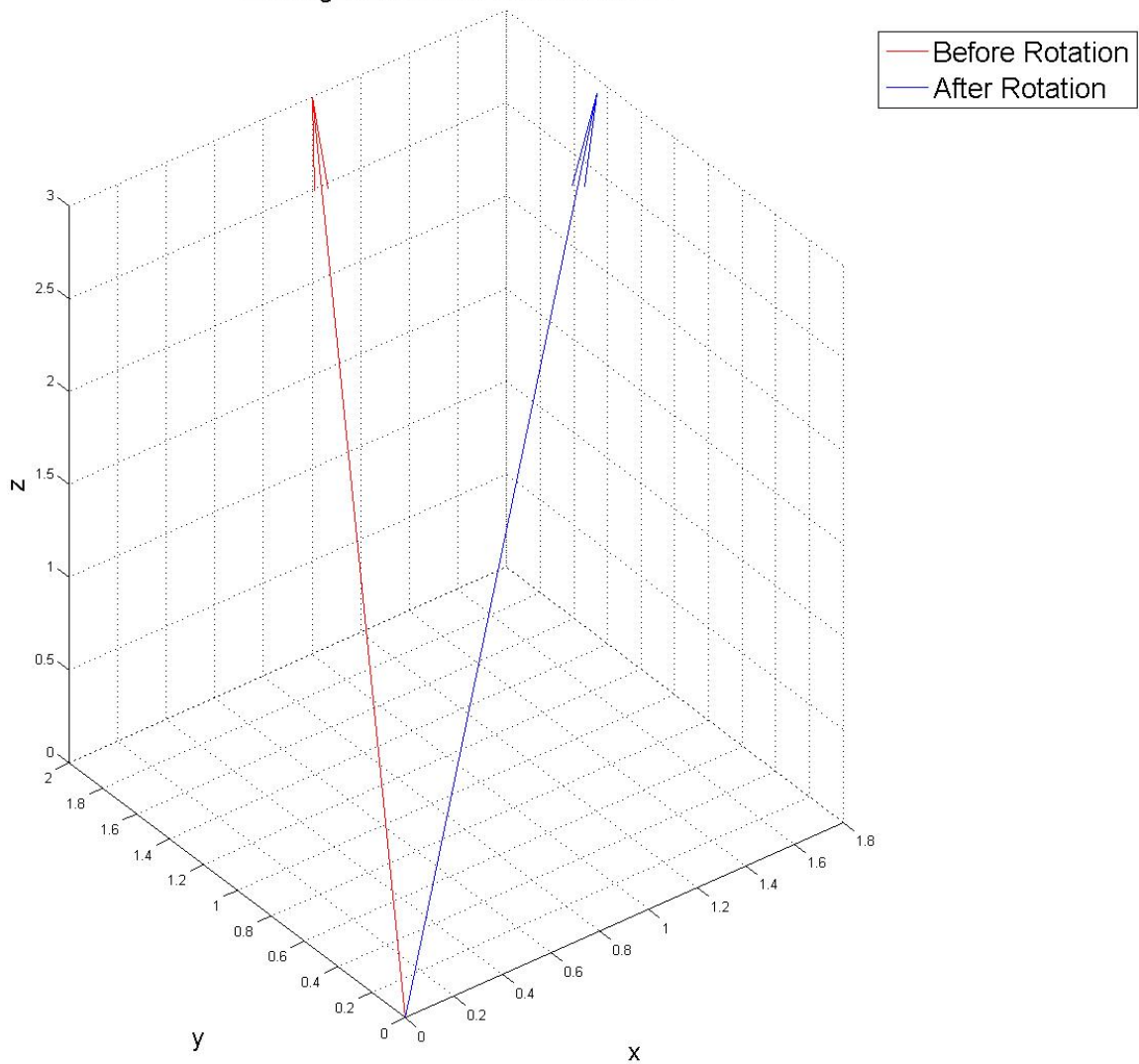


Fig.1 Vector(1,2,3) rotated by 25 degree clockwise about z axis, the magnitude of the vector is 3.74