Ohm's Law

Ohm's law, first presented by German physicist Georg Simon Ohm, states that current is directly proportional to voltage V and inversely proportional to resistance R, or

$$i = \frac{V}{R} \qquad (5\text{-}3)$$

Consider Figure 5-1. If the battery supplies 9 V, and the resistor has a value of 10 Ω, the current measured by the ammeter will be 0.9 amperes. Or, if resistance is increasing, it will take an increasing voltage to maintain the same current.

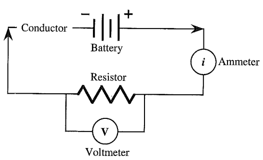


Figure 5-1 A simple electrical circuit illustrating standard symbols for common components.

Resistance and Resistivity

This behavior suggests that the resistances of the resistors in Figure 5-3 depend on their length and cross-sectional areas and also to a fundamental property of the material used in their construction, which we term *resistivity* and denote by ρ. Based on our discussion, we can say that

$$R = \rho \frac{l}{A} \qquad (5\text{-}4)$$

or

$$\rho = R \frac{A}{l} \qquad (5\text{-}5)$$

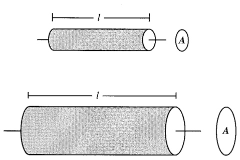


Figure 5-3 Two resistors of different lengths l and different cross-sectional areas A.

Darcy's Law and Ohm's Law

- q = -k(dh/dl)
  - k = hydraulic conductivity
  - h = head
  - l = distance
- i = (1/R)ΔV/L

  ΔV = difference in potential over distance L

  R = Resistance (inverse of conductivity)

The resistivity unit is resistance · length, which is commonly denoted by Ω · m. Conductance is the inverse of resistance, and conductivity is the inverse of resistivity.

> Copper has a resistivity or 1.7 x 10^{-8} Ω • m. What is the resistance of 20 m of copper wire with a cross-sectional radius of 0.005 m? Quartz has a resistivity of 1 x 10^{16} Ω • m. What is the resistance of a quartz wire of the same dimensions?

R = r(l/A) = 1.7e-8*(20/3.14*$(0.005)^2$)

We now use Eq. 5-6 to determine the potential at P_1. In determining the potential at a point, we compare it to the potential at a point infinitely far away, which by convention is arbitrarily defined to equal zero. The most direct way to determine V is to integrate Eq. 5-6 over its distance D to the current electrode to infinity, or

$$V = \int_D^\infty dV = \frac{i\rho}{2\pi}\int_D^\infty \frac{dr}{r^2} = \frac{i\rho}{2\pi D} \qquad (5\text{-}7)$$

(Van Nostrand and Cook, 1966, p. 28). Equation 5-7 is the *fundamental* equation

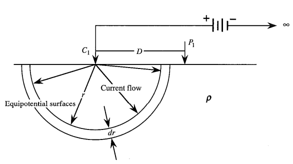


Figure 5-4 Diagram illustrating symbols and configuration used to determine potential at P_1 for a single point source of current C_1. The current sink, C_2, is at infinity. The two equipotential surfaces shown are separated by the distance *dr*.

The potential at point P_1 is determined by using Eq. 5-7. The effect of the source at C_1 (+) and the sink at C_2 (−) are both considered, and, therefore,

$$V_{P_1} = \frac{i\rho}{2\pi r_1} + \left(-\frac{i\rho}{2\pi r_2}\right) \qquad (5\text{-}8)$$

Expressing r_1 and r_2 in terms of the *x-z*-coordinate system illustrated in Figure 5-5, we rewrite Eq. 5-8 as

$$V_{P_1} = \frac{i\rho}{2\pi}\left\{\frac{1}{\left[\left(d/2 + x\right)^2 + z^2\right]^{1/2}} - \frac{1}{\left[\left(d/2 - x\right)^2 + z^2\right]^{1/2}}\right\} \qquad (5\text{-}9)$$

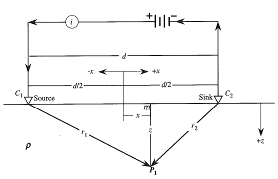


Figure 5-5 Diagram illustrating symbols and configuration used to determine potential at P_1 for a current source C_1 and sink C_2.

Current Penetration is a function of separation of current electrodes

Along a vertical plane midway between the two current electrodes, the fraction of the total current i_f penetrating to depth z for an electrode separation of d is given by

$$i_f = \frac{2}{\pi} \tan^{-1}\left(\frac{2z}{d}\right) \qquad (5\text{-}10)$$

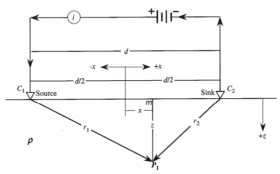


Figure 5-5 Diagram illustrating symbols and configuration used to determine potential at P_1 for a current source C_1 and sink C_2.

TABLE 5-2 Percent Current Penetrating a Homogeneous, Isotropic Earth

Depth (m)	Depth/Electrode Separation	% of Total Current
1	0.1	13
2	0.2	24
3	0.3	34
4	0.4	43
5	0.5	50
6	0.6	56
7	0.7	61
8	0.8	64
9	0.9	68
10	1	70
11	1.1	73
12	1.2	75
13	1.3	77
14	1.4	78
15	1.5	80
16	1.6	81
17	1.7	82
18	1.8	83
19	1.9	84
20	2	84
Current electrode separation (m)		**10**

Current Penetration is a function of separation of current electrodes

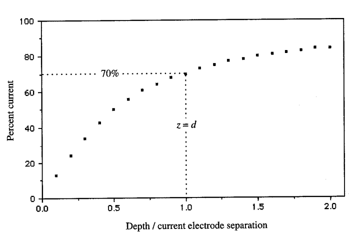


Figure 5-7 Plot of results in Table 5-2. Depth is z and current electrode separation is d. The data points illustrate the extent to which current penetrates into a homogeneous, isotropic Earth.

Figure 5-9 illustrates two potential electrodes P_1 and P_2 that are located on the surface as are the current electrodes. Using the equation we have already derived to determine the potential at a point due to a source and a sink, we obtain the potential difference by determining the potential at one potential electrode P_1 and subtracting from it the potential at P_2. Using Eq. 5-8 we determine that

$$V_{P_1} = \frac{i\rho}{2\pi r_1} - \frac{i\rho}{2\pi r_2} \qquad (5\text{-}11)$$

and

$$V_{P_2} = \frac{i\rho}{2\pi r_3} - \frac{i\rho}{2\pi r_4} \qquad (5\text{-}12)$$

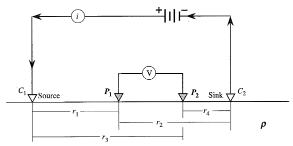


Figure 5-9 Diagram used to determine potential difference at two potential electrodes P_1 and P_2.

Therefore, the potential difference ΔV equals

$$\Delta V = V_{P_1} - V_{P_2} = \left(\frac{i\rho}{2\pi r_1} - \frac{i\rho}{2\pi r_2}\right) - \left(\frac{i\rho}{2\pi r_3} - \frac{i\rho}{2\pi r_4}\right) \qquad (5\text{-}13)$$

or

$$\Delta V = \frac{i\rho}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{r_3} + \frac{1}{r_4}\right) \qquad (5\text{-}14)$$

In the resistivity method, current is entered into the ground, potential difference is measured, and resistivity determined. Because resistivity is the unknown quantity we normally hope to determine, we solve Eq. 5-14 for ρ and obtain

$$\rho = \frac{2\pi\Delta V}{i}\left(\frac{1}{\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{r_3} + \frac{1}{r_4}}\right) \qquad (5\text{-}15)$$

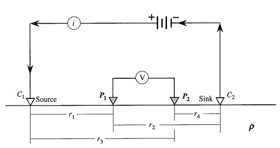


Figure 5-9 Diagram used to determine potential difference at two potential electrodes P_1 and P_2.

Perhaps we should test our understanding of Eq. 5-15 by applying it to a known situation. Let's assume we can place potential electrodes anywhere along the surface, as illustrated in Figure 5-9. Further, we will use the values in Table 5-1 for our test. Figure 5-10(a) presents one possible measurement, and Figure 5-10(b) another. Substituting the values in Figure 5-10(a) produces Eq. 5-16, which results in a resistivity value of 50 Ω · m. A glance at the model values used to produce Table 5-1 confirms that the resistivity is 50 Ω · m.

$$\rho = \frac{2\pi\,(4.5\text{V})}{1\text{ ampere}}\left(\frac{1}{\frac{1}{3\text{ m}} - \frac{1}{7\text{ m}} - \frac{1}{8\text{ m}} + \frac{1}{2\text{ m}}}\right) = 50\ \Omega \cdot \text{m} \qquad (5\text{-}16)$$

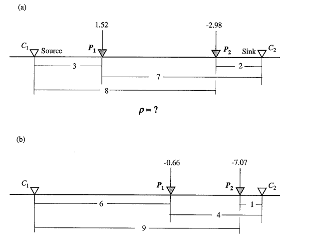


Figure 5-10 Measuring potential difference to determine ρ. Potential values for the indicated spacings are taken from Table 5-1. Solutions are presented in the text (Eqs. 5-16 and 5-17).

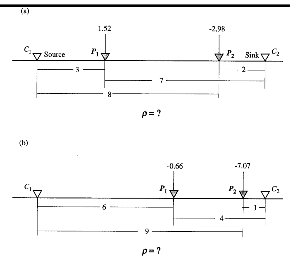


Figure 5-10 Measuring potential difference to determine ρ. Potential values for the indicated spacings are taken from Table 5-1. Solutions are presented in the text (Eqs. 5-16 and 5-17).

Equation 5-17 uses the values illustrated in Figure 5-10(b). As expected, evaluation of the equation gives 50 $\Omega \cdot$ m as the resistivity.

$$\rho = \frac{2\pi\,(6.41\,\mathrm{V})}{1\ \mathrm{ampere}}\left(\frac{1}{\frac{1}{6\ \mathrm{m}} - \frac{1}{4\ \mathrm{m}} - \frac{1}{9\ \mathrm{m}} + \frac{1}{1\ \mathrm{m}}}\right) = 50\ \Omega\cdot\mathrm{m} \qquad (5\text{-}17)$$

These calculations confirm that if we produce a current, measure the electrode spacings, and determine the potential difference, we can arrive at a value for the resistivity of the subsurface materials.

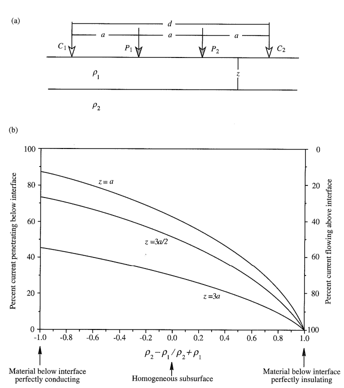
Current Distribution

An important goal in this section is to gain a qualitative understanding for the pattern of current distribution in the subsurface when a single horizontal interface separates materials of different resistivities. Our first step toward this goal is to employ an equation that tells us the fraction of the current that penetrates below the interface. This current fraction is given by

$$i_F = \frac{2\rho_1}{\pi\rho_2}(1+k)\sum_{n=0}^{\infty} k^n \left\{\frac{\pi}{2} - \tan^{-1}\left[\frac{2(2n+1)z}{3a}\right]\right\} \qquad (5\text{-}18)$$

where

$$k = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

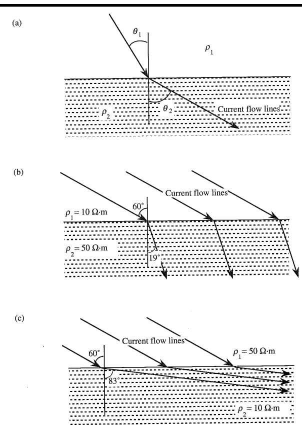


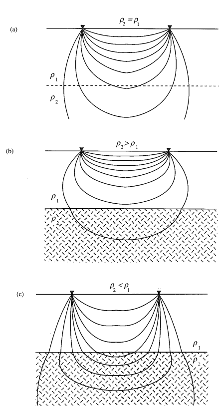
Current Flow Lines and Current Density

The preceding discussion presents us with sufficient information to make a qualitative assessment of current flow lines and, more importantly, current density distribution when a horizontal interface is present. As a first step in this process, we must investigate what happens to the orientation of flow lines and equipotentials when crossing a boundary separating regions of differing conductivities or resistivities. Hubbert (1940, p. 844–846) demonstrated that the flow lines follow a tangent relationship such that

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{\rho_2}{\rho_1} \qquad (5\text{-}19)$$

where θ and ρ are as defined in Figure 5-12(a). If the resistivity ρ_2 of the deeper material is greater, then the flow lines bend in toward the normal to the interface (Fig. 5-12(b)) and, as a consequence, are more widely spaced. However, if the reverse is true, as in Figure 5-12(c), the flow lines bend away from the normal, become oriented more parallel to the interface, and are closer together.





Apparent Resistivity

When we derived Eq. 5-15, we assumed a homogeneous, isotropic subsurface. As demonstrated previously, any combination of electrode spacings and current results in a potential difference that provides the correct value for the resistivity of the subsurface (as of course should be the case if our equation is correct). Once the subsurface is nonhomogeneous, the value determined for the resistivity is extremely unlikely to equal the resistivity of the material in which the electrodes are inserted. Equation 5-15 thus defines a different quantity, which is termed the *apparent resistivity* ρ_a. Inasmuch as nonhomogeneity is the rule, we write

$$\rho_a = \frac{2\pi\Delta V}{i}\left(\frac{1}{\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{r_3} + \frac{1}{r_4}}\right) \qquad (5\text{-}20)$$

The question we now face is, What does this equation tell us? How do we interpret apparent resistivity values in terms of the subsurface geology?

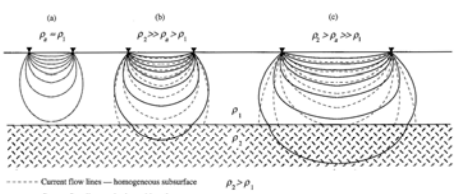
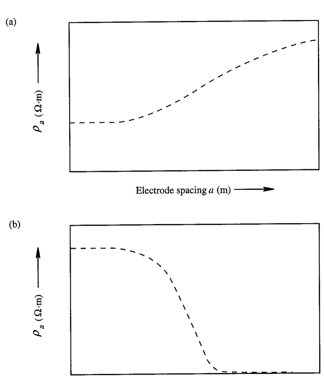
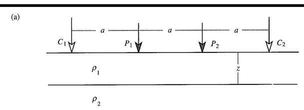


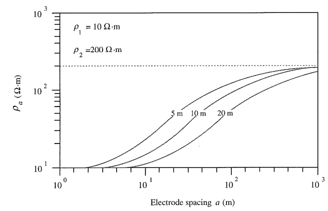
Figure 5-14 Effect on apparent resistivity as electrode spacing is increased. Dashed black lines represent current flow line distribution for homogeneous subsurface. Solid lines represent actual current flow lines due to horizontal interface. As the distance between current electrodes is increased, ρ_a approaches the value of ρ_2.





ρ_1 (Ω • m)	ρ_2 (Ω • m)	Depth to Interface
10	200	5
10	200	10
10	200	20

The important aspect of these curves is their similarities and differences, not the absolute values of the apparent resistivities and *a*-spacings.



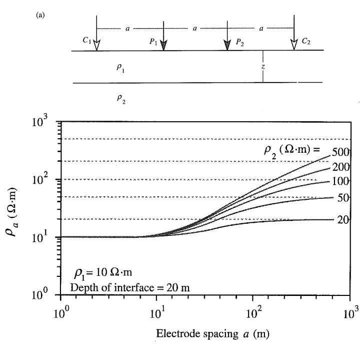
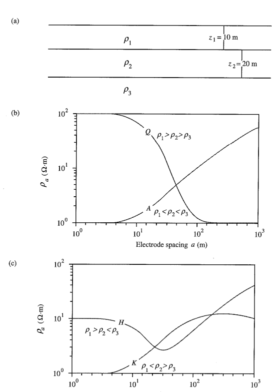


Figure 5-20 Apparent resistivity curves for constant depth to an interface. Resistivity of the layer above the interface also is constant. Dashed lines are drawn at resistivity values assigned to the layer beneath the interface. Pay special attention to the different curve forms for the various resistivities assigned to the deeper layer.

3 Layer Models



Suppression and Equivalence

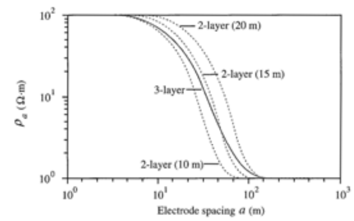
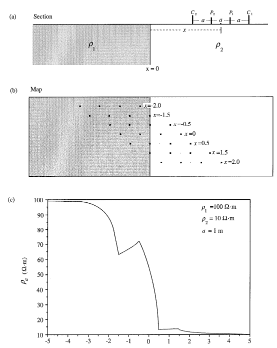


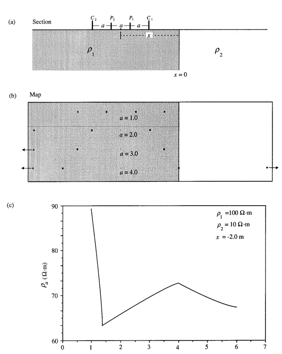
Figure 5-22 Comparison of two- and three-layer curves for similar depths and resistivities. The three-layer curve is a solid line (thickness of the first layer is 10 m; thickness of the second layer is 20 m; resistivities are 100, 10, and 1 $\Omega \cdot$ m). Resistivities for the two-layer cases are 100 and 1 $\Omega \cdot$ m. Depth of the interface is indicated for each curve.

Horizontal Survey across a Vertical Interface

Why is the change in apparent resistivity values much more pronounced between $x = -3$ to $x = -2$ than from $x = 3$ to $x = 2$? *Hint:* Note that the negative x-values are in the higher resistivity medium. Use the current density distribution model or a water-flow analogy to arrive at a qualitative explanation.



Vertical Survey across a Vertical Interface



Vertical Dike

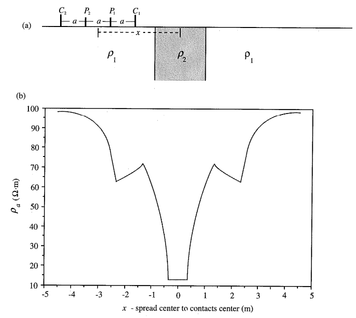


Figure 5-25 Constant-spread traverse across a vertical dike. (a) Diagram of relationships. (b) Qualitatively derived sketch of likely curve shape.

Channel is Similar to Vertical Dike Model

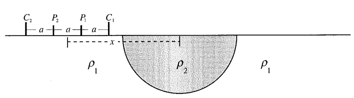


Figure 5-26 Hemispherical sink with diameter substantially greater than electrode spacing *a*.

Field Methods

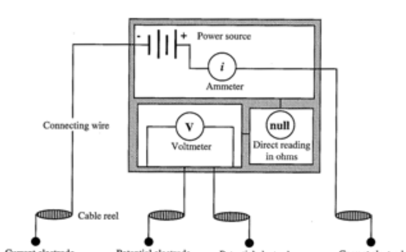
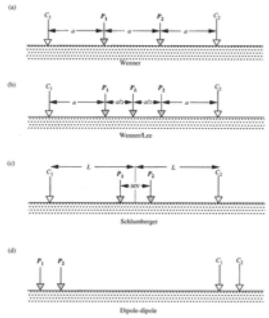


Figure 5-28 Schematic of main elements of electrical-resistivity surveying system including electrodes, power source, ammeter, voltmeter, and direct-reading device.

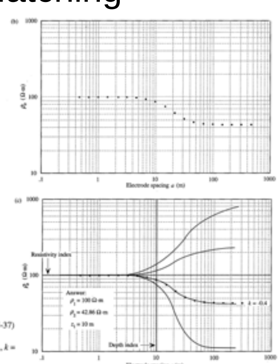
Field Methods



Resistivity of Earth Materials

Material	Resistivity (Ω·m)
Wet to moist clayey soil and wet clay	1s to 10s
Wet to moist silty soil and silty clay	Low 10s
Wet to moist silty and sandy soils	10s to 100s
Sand and gravel with layers of silt	Low 1000s
Coarse dry sand and gravel deposits	High 1000s
Well-fractured to slightly fractured rock with moist-soil-filled cracks	100s
Slightly fractured rock with dry, soil-filled cracks	Low 1000s
Massively bedded rock	High 1000s

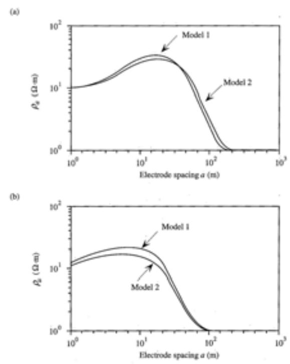
Curve Matching



$$\rho_2 = \left(\frac{1+k}{1-k}\right)\rho_1 \qquad (5\text{-}37)$$

it is straightforward to determine ρ_2. Figure 5-33(c) provides values of $\rho_1 = 100\ \Omega\cdot\text{m}$, $k = -0.4$, and $z_1 = 10$ m. Equation (5-37) then provides a value of 42.86 Ω·m for ρ_2.

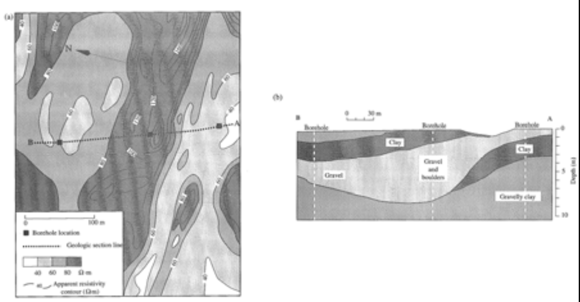
Equivalence and Suppression



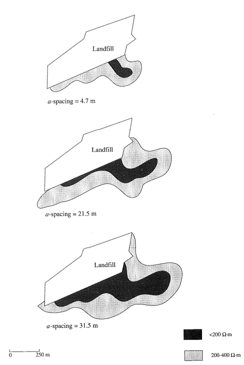
Model 1 – 3 m, 10 Ohm-m
10 m, 100 Ohm-m
inf., 1 Ohm-m
Model 2 - 3 m, 10 Ohm-m
20 m, 50 Ohm-m
inf., 1 Ohm-m

Model 1 – 1 m, 10 Ohm-m
1 m, 40 Ohm-m
1 m, 10 Ohm-m
7 m, 40 Ohm-m
inf., 1 Ohm-m
Model 2 - 1 m, 10 Ohm-m
13 m, 20 Ohm-m
inf., 1 Ohm-m

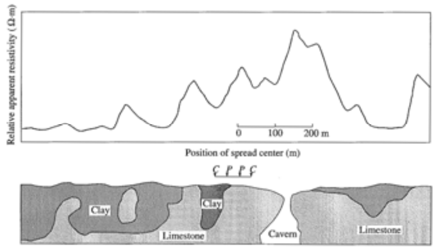
3D survey of Stream Channel (6.1 m spacing)



Leachate Plumes



Air Filled Cavern/Limestone (a = 30.48 m)



Horizontal Survey of Faults (a = 30.48 m)

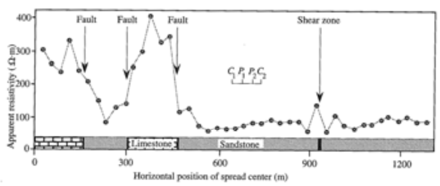


Figure 5-39 Apparent resistivity curve for a constant-spread traverse over faults in Illinois. Wenner array, *a*-spacing = 30.48 m, spread-center spacing = 30.48 m. Curve form is similar to that predicted over vertical contacts. (Based on data in Hubbert, M. K., 1932, Results of earth-resistivity survey on various geologic structures in Illinois: American Institute of Mining and Metallurgical Engineers Technical Publication 463, p. 16.)