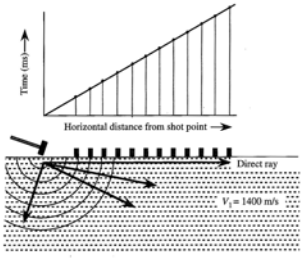
Direct Waves – Velocity from x-t



$$\text{time} = \frac{\text{path}}{\text{velocity}} \qquad (3\text{-}1)$$

$$t = \frac{x}{V_1} \qquad (3\text{-}2)$$

$$\frac{dt}{dx} = \frac{1}{V_1} \qquad (3\text{-}3)$$

$$\text{slope} = \frac{1}{V_1} \qquad (3\text{-}4)$$

Direct Wave Arrivals

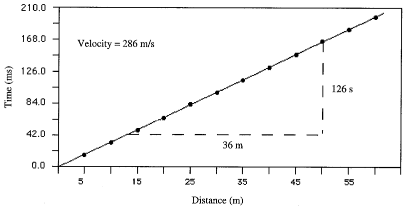


Figure 3-2 Travel-time curve (time-distance graph) illustrating only direct wave arrivals. The velocity is 286 m/s as determined from the inverse of the slope.

$$V_1 = \frac{1}{\text{slope}} \qquad (3\text{-}5)$$

Head Wave Distance vs. Time

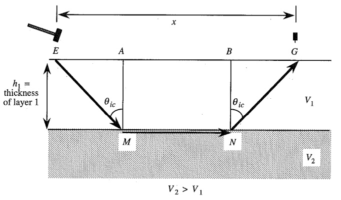


Figure 3-3 Diagram illustrating symbols used in derivation of time of travel for critically refracted ray.

The total time of travel must be

$$\text{time} = \frac{EM}{V_1} + \frac{MN}{V_2} + \frac{NG}{V_1} \qquad (3\text{-}6)$$

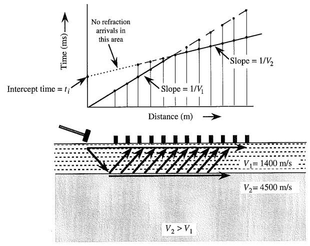


Figure 3-4 Generalized diagram illustrating ray paths in a material with one horizontal discontinuity. Time-distance relationships for both the direct and refracted rays are shown in the travel-time curve.

$$\text{time} = \frac{2h_1(V_2^2 - V_1^2)^{1/2}}{V_2V_1} + \frac{x}{V_2} \qquad (3\text{-}14)$$

With the final form of our travel-time equation (3-14) directly in front of us, let's follow the same procedure as we did for the direct wave and take the first derivative of the equation with respect to x. This provides a perhaps surprisingly simple result:

$$\frac{dt}{dx} = \frac{1}{V_2} \qquad (3\text{-}15)$$

Thickness from Intercept Time

Determining Thickness

Examine Figure 3-4 once again. The straight line passing through the arrival times for the critically refracted ray can be extended until it intersects the time axis. This time is termed the *intercept time* t_i. Recall that it has no real physical significance, because no refractions arrive at the energy source ($x = 0$). However, at $x = 0$ our travel-time Eq. 3-14 reduces to

$$\text{time} = t_i = 2h_i\frac{(V_2^2 - V_1^2)^{1/2}}{V_2V_1} \qquad (3\text{-}16)$$

and, therefore,

$$h_1 = \frac{t_i}{2}\,\frac{V_2V_1}{(V_2^2 - V_1^2)^{1/2}} \qquad (3\text{-}17)$$

Thus, for a single horizontal interface, if we can determine times for direct and refracted arrivals from a field seismogram, we can calculate the thickness of the material above the interface and the velocities of the materials above and below the interface.

Thickness from Crossover Distance

At x_{co} the times of travel for the direct wave and the refracted wave are equal, and, therefore,

$$\text{time}_{\text{direct wave}} = \frac{x_{co}}{V_1} \qquad \text{(see 3-2)}$$

$$\text{time}_{\text{refracted wave}} = \frac{2h_1(V_2^2 - V_1^2)^{1/2}}{V_2V_1} + \frac{x_{co}}{V_2} \qquad \text{(see 3-14)}$$

and

$$\frac{x_{co}}{V_1} = \frac{2h_1(V_2^2 - V_1^2)^{1/2}}{V_2V_1} + \frac{x_{co}}{V_2} \qquad (3\text{-}18)$$

If we rearrange terms, we arrive at

$$\frac{h_1(V_2^2 - V_1^2)^{1/2}}{V_2V_1} = \frac{x_{co}}{2}\left(\frac{V_2 - V_1}{V_2V_1}\right) \qquad (3\text{-}19)$$

and

$$h_1 = \frac{x_{co}}{2}\left(\frac{V_2 - V_1}{V_2V_1}\right)\left(\frac{V_2V_1}{(V_2^2 - V_1^2)^{1/2}}\right) \qquad (3\text{-}20)$$

The V_1V_2 terms cancel, and remembering that $(V_2^2 - V_1^2) = (V_2 - V_1)(V_2 + V_1)$, we can simplify Eq. 3-20 to

$$h_1 = \frac{x_{co}}{2}\left(\frac{V_2 - V_1}{V_2 + V_1}\right)^{1/2} \qquad (3\text{-}21)$$

It is mainly a matter of convenience whether we use Eq. 3-21 or Eq. 3-17 to determine layer thickness. However, because determination of x_{co} involves fitting *two* lines to field data, whereas t_i requires only one line, t_i usually can be determined with more accuracy.

Critical Distance (no head waves)

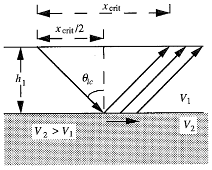


Figure 3-5 This diagram illustrates the relationships for calculating critical distance. Critical distance is the minimum distance from the energy source at which the first critical refraction can be received.

Picking Arrivals on a Seismogram

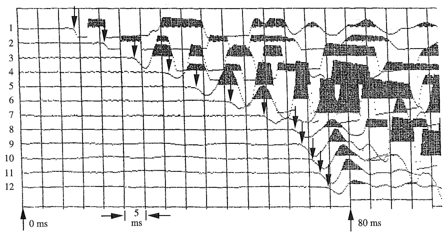


Figure 3-6 Field seismogram from the Connecticut Valley, Massachusetts. Geophone traces are labeled 1–12. The first geophone is located 5 m from the energy source. The geophone interval is 3 m. First breaks for each trace are indicated by a downward directed arrow. Timing lines are at 5-ms intervals. The record encompasses 100 ms. This seismogram exhibits a classic two-layer pattern.

Seismogram Data

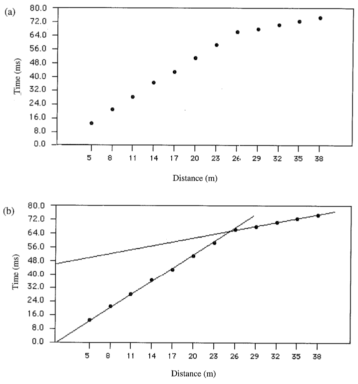


Figure 3-7 (a) Time-distance data from the seismogram in Figure 3-6. (b) Lines drawn through data points in (a).

Mohorovicic Discontinuity

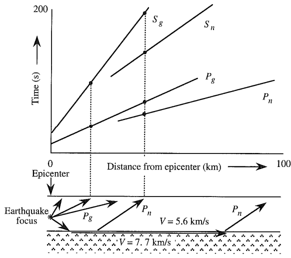


Figure 3-8 Travel-time curves simplified from Mohorovičić's original plots. P_g is the direct compressional wave, and P_n is the refracted compressional wave. Terminology for shear waves S is similar. The section below the travel-time curve illustrates Mohorovičić's interpretation of the travel-time curves.

Multiple Horizontal Layers

We know from Eq. 2-25 (Snell's law) that

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{V_1}{V_2}$$

and, since $\theta_r = \theta_{ic}$, then

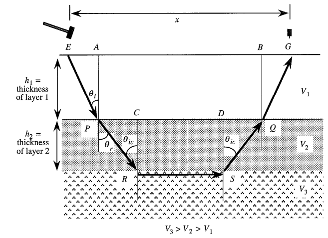


Figure 3-9 Diagram illustrating symbols used in derivation of time of travel for ray critically refracted along the second interface in a three-layer case.

2 Interface Example

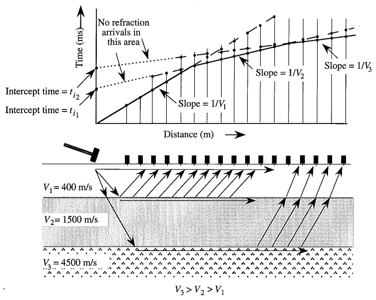


Figure 3-10 Generalized diagram illustrating ray paths in a material with two horizontal interfaces. Time-distance relationships for the direct and two critically refracted rays are shown in the travel-time curve.

2 Interface x-t Plot – computed from RefractModel

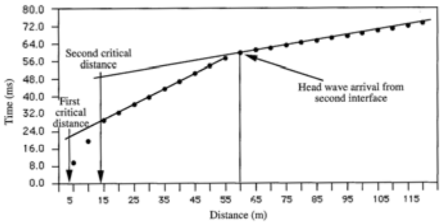


Figure 3-12 Travel-time curve based on a RefractModel plot using velocity values in Table 3-4 with $h_1 = 5$ m and $h_2 = 20$ m. Note that the first refraction from the second interface to be a first arrival is located at a distance considerably greater than the second critical distance.

Field Seismogram

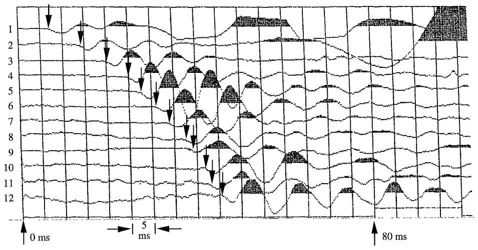


Figure 3-13 Field seismogram from the Connecticut Valley, Massachusetts. Geophone traces are labeled 1–12. The first geophone is located 3 m from the energy source. The geophone interval is 10 m. First breaks for each trace are indicated by a downward directed arrow. Timing lines are at 5-ms intervals. The record encompasses 100 ms.

2 Horizontal Interface x-t Plot – from seismogram (note: just 1 direct wave first arrival)

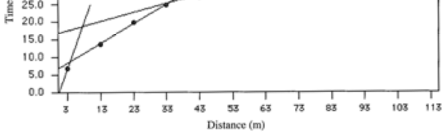


Figure 3-14 Preferred interpretation of time-distance data taken from Figure 3-13.

Horiz. Interface -> Sym. x-t plots

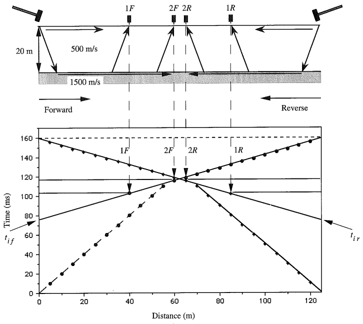


Figure 3-16 Correlation of a travel-time curve with wave paths to geophones at equal distances from an energy source for a forward and a reverse traverse.

Dipping Interface: Velocity <u>not</u> equal to 1/slope

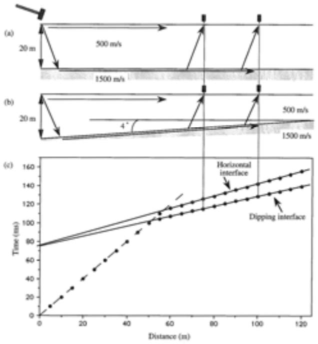


Figure 3-15 (a) Horizontal interface at a depth of 20 m with velocities above and below the interface of 500 m/s and 1500 m/s, respectively. (b) A dipping interface with identical depth to that in (a) at the site of the hammer impact and with identical velocities to (a). (c) Travel-time curves for (a) and (b).

Horizontal Interface: Symmetric x-t

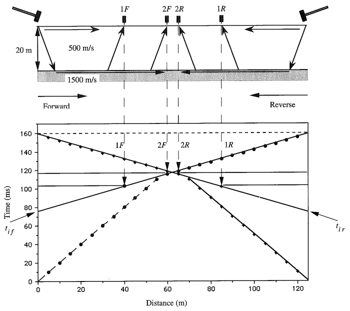


Figure 3-16 Correlation of a travel-time curve with wave paths to geophones at equal distances from an energy source for a forward and a reverse traverse.

Dipping Interface: asymmetric x-t

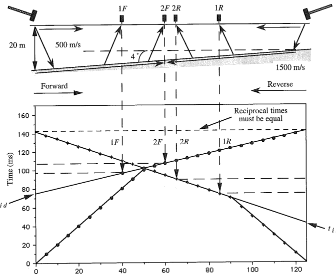


Figure 3-17 Correlation of a travel-time curve with geophone positions above a single dipping interface. The purpose of this diagram is to demonstrate the different path distances and arrival times for geophones located at identical offsets for a forward and reverse traverse.

Intercept Time gives up-dip direction

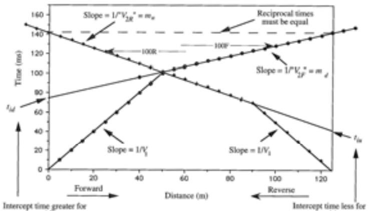
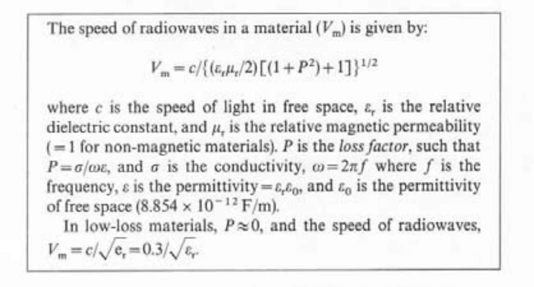


Figure 3-18 Important features of a travel-time curve for a single dipping interface. Note that reciprocal times must be equal. However, the times from forward and reverse shot points to a geophone at a given distance are not equal. This is illustrated by the arrows labeled 100F and 100R, which designate travel times to geophones located 100 m from the forward (F) and reverse (R) shots.

Velocity = Frequency x Wavelength
Frequency = 10 to 1000 MHz
Wavelength = cm to meters



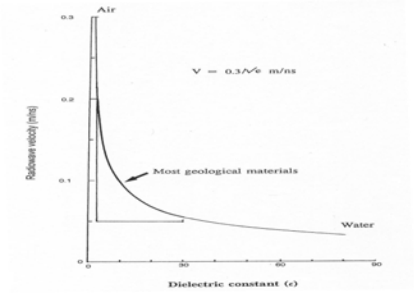
The speed of radiowaves in a material (V_m) is given by:

$$V_m = c/\{(\varepsilon_r \mu_r/2)[(1+P^2)+1]\}^{1/2}$$

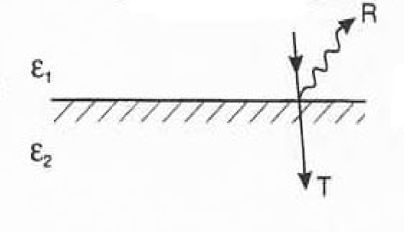
where c is the speed of light in free space, ε_r is the relative dielectric constant, and μ_r is the relative magnetic permeability ($=1$ for non-magnetic materials). P is the *loss factor*, such that $P=\sigma/\omega\varepsilon$, and σ is the conductivity, $\omega=2\pi f$ where f is the frequency, ε is the permittivity $=\varepsilon_r\varepsilon_0$, and ε_0 is the permittivity of free space (8.854×10^{-12} F/m).

In low-loss materials, $P \approx 0$, and the speed of radiowaves, $V_m = c/\sqrt{e_r} = 0.3/\sqrt{\varepsilon_r}$.

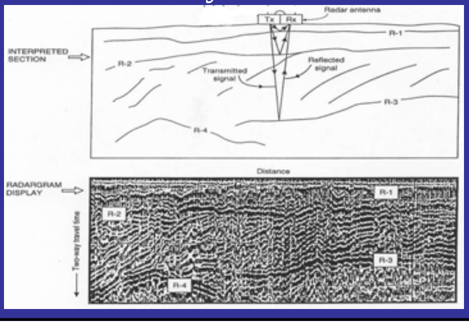
Velocities of Air, Water, and Earth Materials



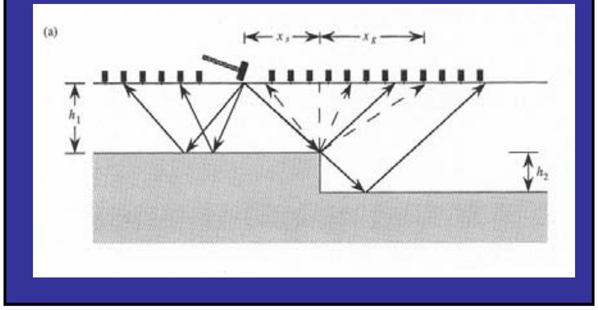
Reflection and Transmission



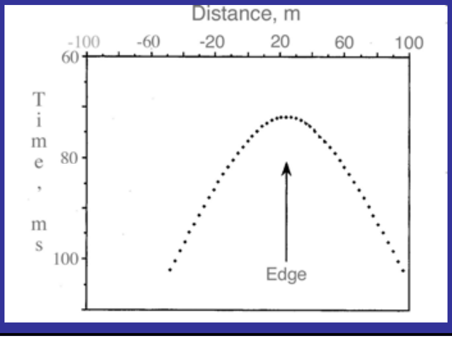
Radargrams - record time that energy reflected back to surface at many locations



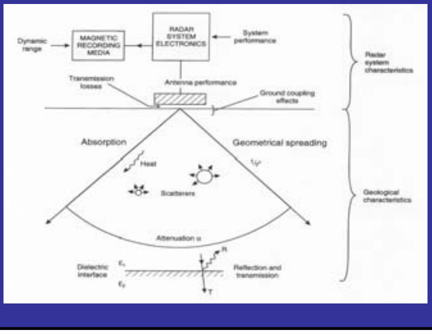
Diffractions off sudden change in interface of same dimensions as wavelength of radar waves



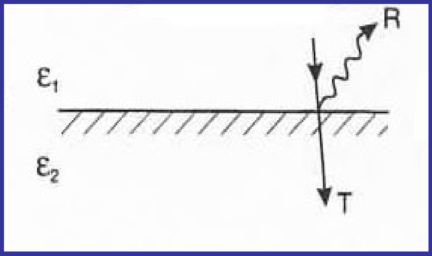
Diffractions appear on Radargram as a Hyperbola



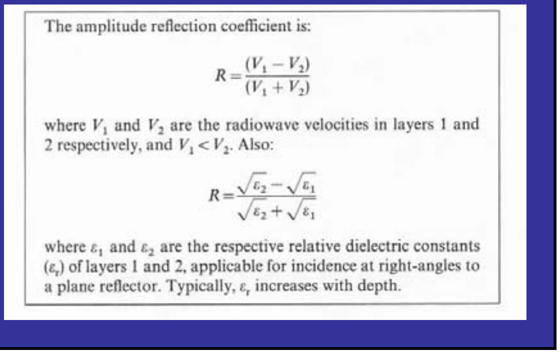
Wave Attenuation and Amplitude - Spherical Spreading and Absorption



Energy Partitioning



Amplitude of Reflection



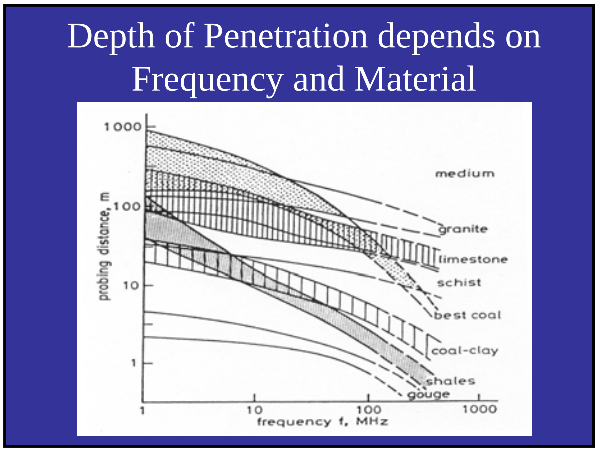
The amplitude reflection coefficient is:

$$R = \frac{(V_1 - V_2)}{(V_1 + V_2)}$$

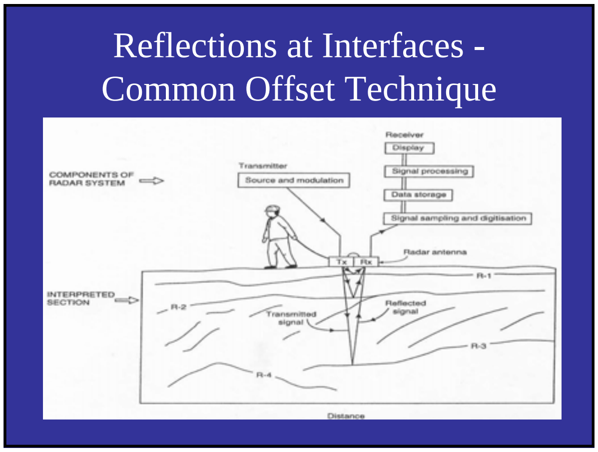
where V_1 and V_2 are the radiowave velocities in layers 1 and 2 respectively, and $V_1 < V_2$. Also:

$$R = \frac{\sqrt{\varepsilon_2} - \sqrt{\varepsilon_1}}{\sqrt{\varepsilon_2} + \sqrt{\varepsilon_1}}$$

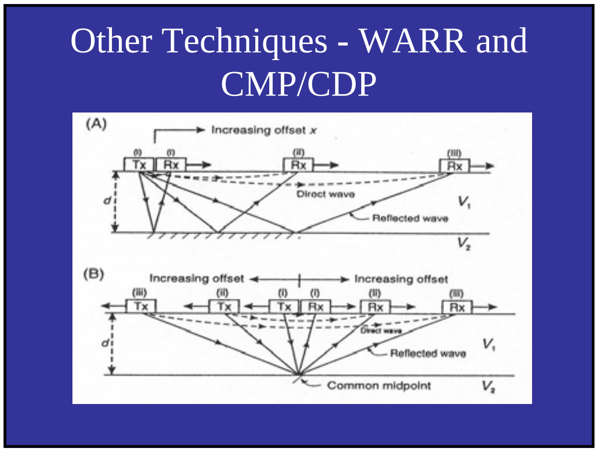
where ε_1 and ε_2 are the respective relative dielectric constants (ε_r) of layers 1 and 2, applicable for incidence at right-angles to a plane reflector. Typically, ε_r increases with depth.



Depth of Penetration depends on Frequency and Material



Reflections at Interfaces - Common Offset Technique



Other Techniques - WARR and CMP/CDP

Dielectric Constant & Velocity

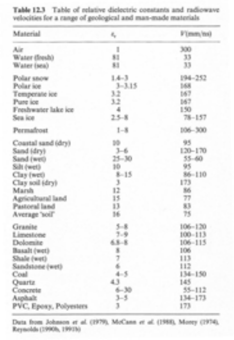


Table 12.3 Table of relative dielectric constants and radiowave velocities for a range of geological and man-made materials

Material	ε_r	V(mm/ns)
Air	1	300
Water (fresh)	81	33
Water (sea)	81	33
Polar snow	1.4–3	194–252
Polar ice	3–3.15	168
Temperate ice	3.2	167
Pure ice	3.2	167
Freshwater lake ice	4	150
Sea ice	2.5–8	78–157
Permafrost	1–8	106–300
Coastal sand (dry)	10	95
Sand (dry)	3–6	120–170
Sand (wet)	25–30	55–60
Silt (wet)	10	95
Clay (wet)	8–15	86–110
Clay soil (dry)	3	173
Marsh	12	86
Agricultural land	15	77
Pastoral land	13	83
Average 'soil'	16	75
Granite	5–8	106–120
Limestone	7–9	100–113
Dolomite	6.8–8	106–115
Basalt (wet)	8	106
Shale (wet)	7	113
Sandstone (wet)	6	112
Coal	4–5	134–150
Quartz	4.3	145
Concrete	6–30	55–112
Asphalt	3–5	134–173
PVC, Epoxy, Polyesters	3	173

Data from Johnson *et al.* (1979), McCann *et al.* (1988), Morey (1974), Reynolds (1990b, 1991b)

Resolution depends on frequency and the material

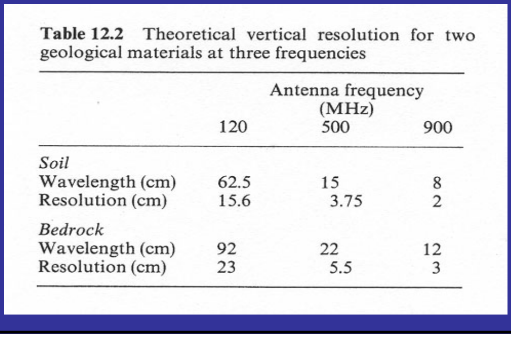
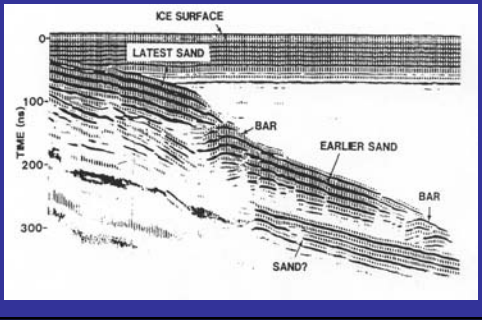
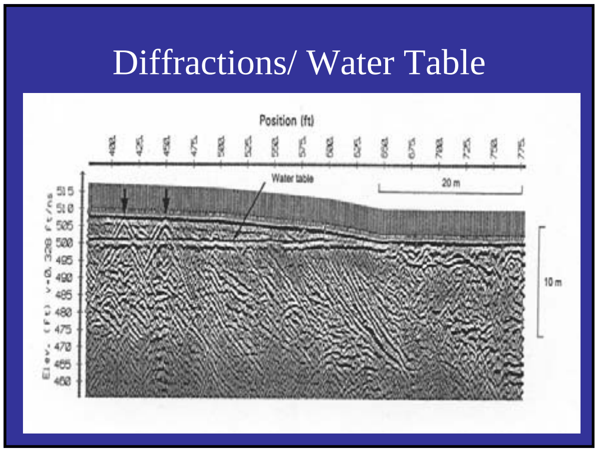


Table 12.2 Theoretical vertical resolution for two geological materials at three frequencies

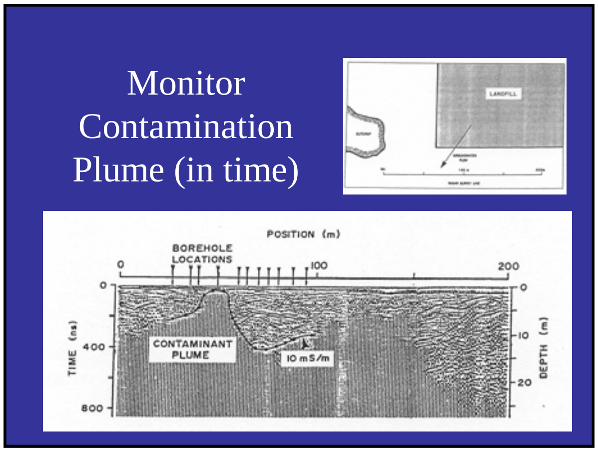
	Antenna frequency (MHz)		
	120	500	900
Soil			
Wavelength (cm)	62.5	15	8
Resolution (cm)	15.6	3.75	2
Bedrock			
Wavelength (cm)	92	22	12
Resolution (cm)	23	5.5	3

Imaging Sediments through Ice and Water

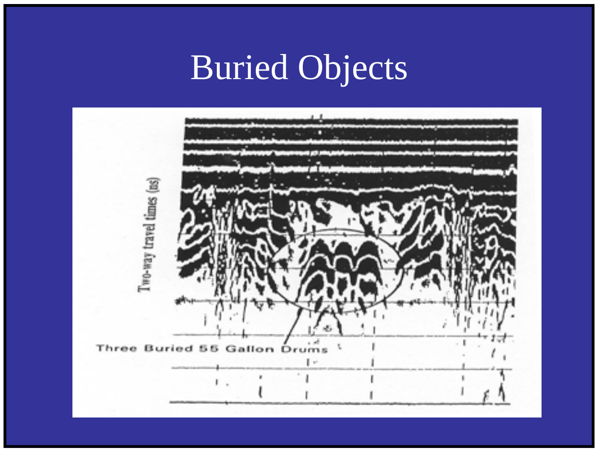




Diffractions/ Water Table

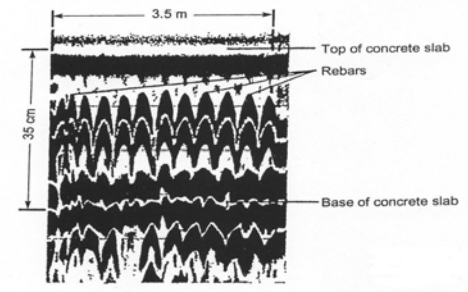


Monitor Contamination Plume (in time)

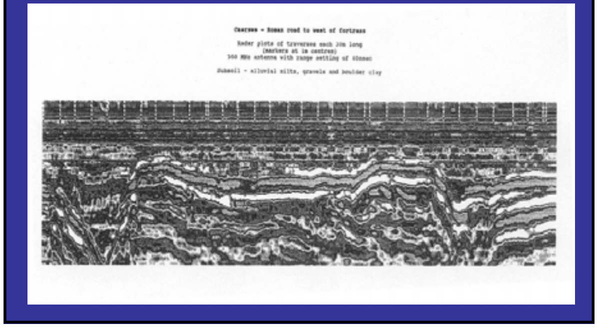


Buried Objects

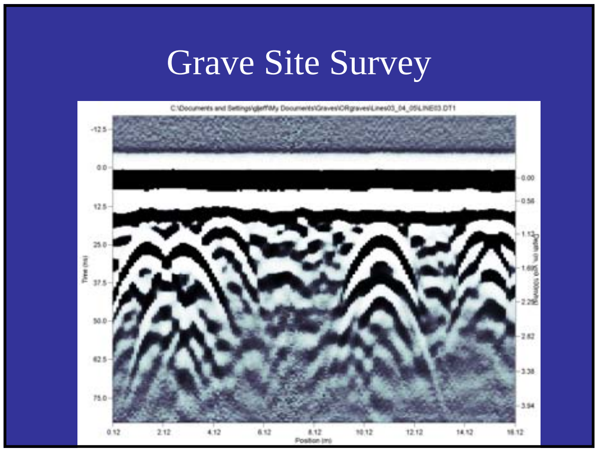
Rebar in Concrete Slab - High Frequency Radar Diffractions



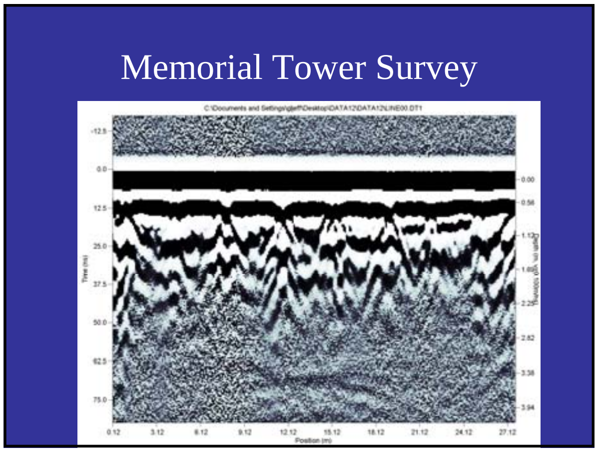
Buried Roman Road with ditches and cart ruts



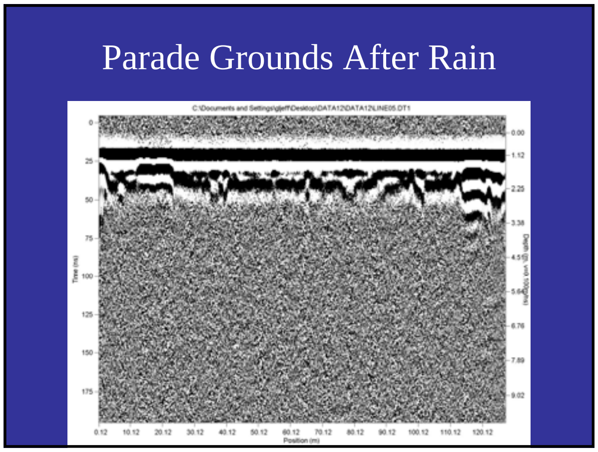
Grave Site Survey



Grave Site Survey



Memorial Tower Survey



Parade Grounds After Rain

Lacoste-Romberg Gravimeter
Hooke's Law (Strain proportional to Stress)

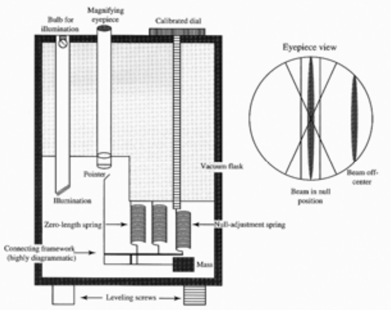


FIGURE 6.1 A highly diagrammatic representation of the interior of a Worden gravimeter.

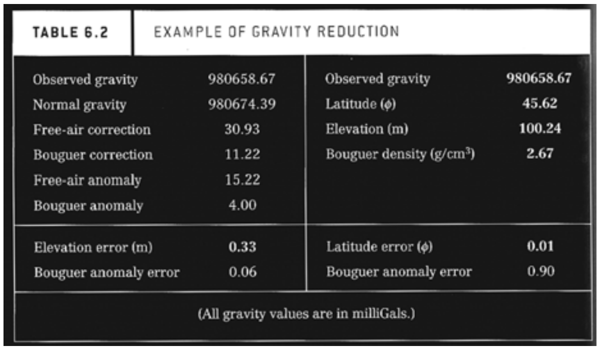


TABLE 6.2 EXAMPLE OF GRAVITY REDUCTION

Observed gravity	980658.67	Observed gravity	**980658.67**
Normal gravity	980674.39	Latitude (ϕ)	**45.62**
Free-air correction	30.93	Elevation (m)	**100.24**
Bouguer correction	11.22	Bouguer density (g/cm^3)	**2.67**
Free-air anomaly	15.22		
Bouguer anomaly	4.00		
Elevation error (m)	**0.33**	Latitude error (ϕ)	**0.01**
Bouguer anomaly error	0.06	Bouguer anomaly error	0.90

(All gravity values are in milliGals.)

Gravity From a Buried Sphere

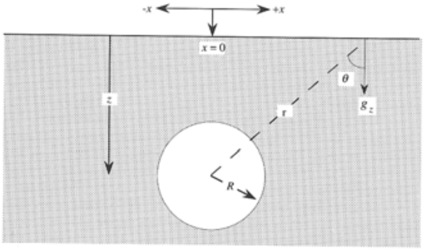
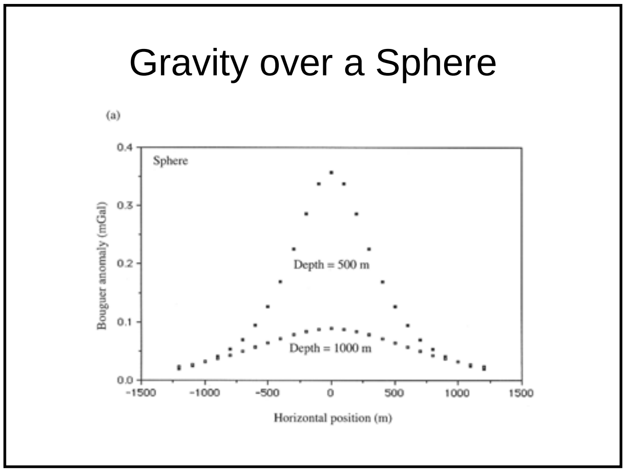
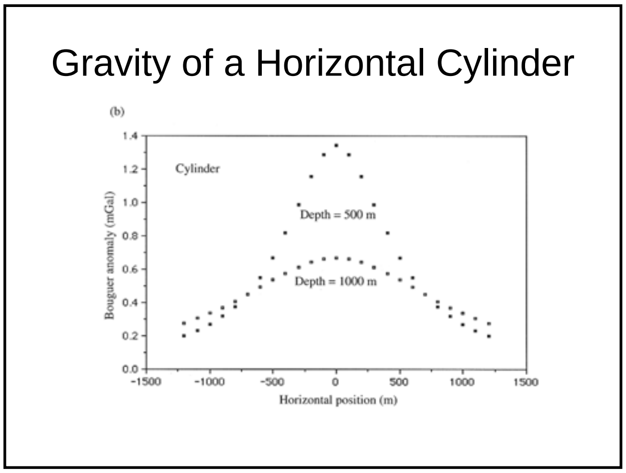


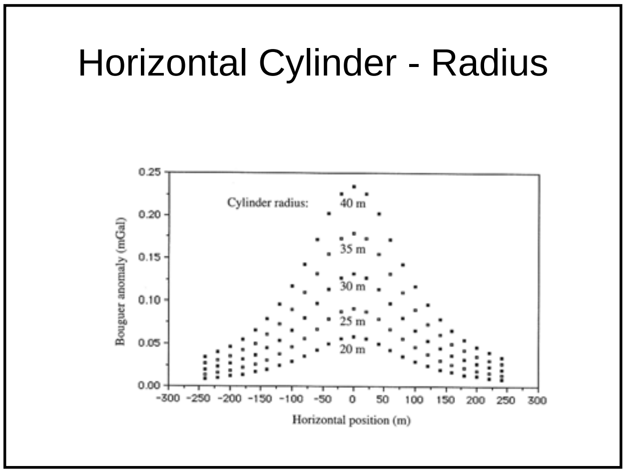
Figure 6-13 Notation used in derivation of the gravity effect of a buried sphere. The same notation is used for a traverse at right angle to the strike of a horizontal, infinitely long cylinder.



Gravity over a Sphere

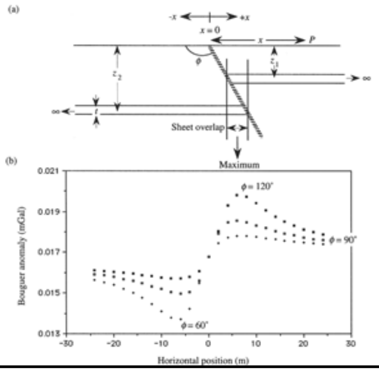


Gravity of a Horizontal Cylinder



Horizontal Cylinder - Radius

Reverse Fault



Normal Fault