Ground-Water Flow to Wells

Introduction

Wells used to control salt water intrusion, remove contaminated water, lower the water table for construction, relieve pressure under dams, and drain farmland

Determine the drawdown or cone of depression if transmissivity and storativity are known or vice versa

Assumptions:

full penetration of aquifer,
radial flow,
homogeneous and isotropic,
steady state system prior to pumping,
constant density and viscosity of water,
pumping well of infinitesimal diameter and 100% efficient
horizontal flow and Darcy's law valid
potentiometric surface initially horizontal and any changes due to pumping of well,
aquifer horizontal, infinite in horizontal extent, and bounded on bottom by confining layer

Drawdown Caused by a Pumping Well

Unsteady Radial Flow

In polar coordinates, the equation for confined flow is

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{T}\frac{\partial h}{\partial t}$$

where h = hydraulic head
S = storativity
T = transmissivity
t = time
r = radial distance from pumping well

If there is leakage through a confining layer

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} + \frac{e}{T} = \frac{S}{T}\frac{\partial h}{\partial t}$$

where e = rate of vertical leakage

Flow in a Completely Confined Aquifer

Assumptions - aquifer confined both top and bottom
no source of recharge
aquifer is compressible (can store water)
well pumps at constant rate

Theis equation

$$h_0 - h = \frac{Q}{4\pi T}\left[-0.5772 - \ln u + u - \frac{u^2}{2x2!} + \frac{u^3}{3x3!} - ... \right]$$

$$= \frac{Q}{4\pi T} w(u)$$

where $u = \frac{r^2 S}{4Tt}$

$h_0 - h$ = the drawdown
Q = pumping rate
t = time since pumping began

Flow in a Leaky, Confined Aquifer

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} + \frac{(h_0 - h)K'}{Tb'} = \frac{S}{T}\frac{\partial h}{\partial t}$$

Solution 1 - No water drains from the Confining Layer

Assumptions - aquifer bounded on top by aquitard and unconfined aquifer (source bed);
water table in source bed initially horizontal and does not change during pumping;
aquitard is incompressible and flow through it is vertical;
aquifer is compressible (stores water)

Water table in source bed unchanged if

$$t < \frac{S'(b')^2}{10bK'}$$ (time of pumping is short)

$b''K'' > 100\ bK$ (source bed is large reservoir compared to aquifer)

Aquitard is incompressible if

$t > 0.036b'S'/K'$ (time of pumping is large relative to amount of water in storage)

$r < 0.04b[(KS_s/K'S'_s)]^{1/2}$

Solution valid if well radius, r_w, meets the following 2 conditions:

$t > (30r_w{}^2S/T)\ [1 - (10r_w/b)^2]$

$r_w/(Tb'/K')^{1/2} < 0.1$

Hantush-Jacob formula

$$h_0 - h = \frac{Q}{4\pi T}\ W(u,r/B)$$

where $u = \dfrac{r^2S}{4Tt}$

$B = (Tb'/K')^{1/2}$ = leakage factor

W = leaky Artesian well function

Rate at which water is being drawn from elastic storage, q_s, at a specific time is

$$q_s = Q \exp(-Tt/SB^2)$$

If the well is pumped long enough all of the water will be coming from leakage across the confining layer, this occurs when

$$t > \frac{8b'S}{K'}$$

and drawdown is found from

$$h_0 - h = \frac{Q}{2\pi T} K_0(r/B)$$

where K_0 = zero-order modified Bessel function

Solution 2 - Water drains from the Confining Layer

Early period of pumping

$$t > b'S'/10K'$$

$$h_0 - h = \frac{Q}{4\pi T} H(u,\beta)$$

where $\beta = \frac{r}{4B} (S'/S)^{1/2}$

$$B = \left(\frac{Tb'}{K'}\right)^{1/2}$$

$$q_s = Q \exp(vt) \operatorname{erfc}((vt)^{1/2})$$

where $v = (K'/b')(S'/S^2)$

If sufficient time elapses, the aquifer will reach equilibrium (all of the water will be coming from the source bed)

$$t > \frac{8[s + (S'/3) + S'']}{[(K'/b'T) + K''/b''T}]^{1/2})$$

if $r_w/B < 0.01$, then $h_0 - h = \frac{Q}{2\pi T} K_0(r/B)$

Flow in an Unconfined Aquifer

$$K_r \frac{\partial^2 h}{\partial r^2} + \frac{K_r}{r}\frac{\partial h}{\partial r} + K_v \frac{\partial^2 h}{\partial z^2} = S_s \frac{\partial h}{\partial t}$$

where h = saturated thickness
z = elevation above the base of the aquifer
S_s = specific storage
K_v = vertical hydraulic conductivity
K_r = radial hydraulic conductivity

water extracted by 2 mechanisms - elastic storage and specific yield

3 distinct phases of drawdown:

1) early - water released from elastic storage, follows Theis nonequilibrium curve, flow is horizontal

2) middle - water table begins to decline, water derived from gravity drainage, 3D flow, drawdown controlled by anisotropy of aquifer and thickness of aquifer

3) late - rate of drawdown decreases, flow is horizontal, and follows Theis curve with storativity equal to specific yield

Neuman's solution -
unconfined aquifer, vadose zone not important
initial water comes from specific storage
eventually water comes from gravity drainage
drawdown small compared to saturated thickness
specific yield is at least ten times elastic storativity

$$h_0 - h = \frac{Q}{4\pi T} W(u_A, u_B, \Gamma)$$

where $u_A = \frac{r^2 S}{4Tt}$

$$u_B = \frac{r^2 S_y}{4Tt}$$

$$\Gamma = \frac{r^2 K_v}{b^2 K_h}$$

Determining Aquifer Parameters from Time-Drawdown Data

Aquifer Test Assumptions

pumping well screened only in aquifer being tested
observation wells screened only in aquifer being tested
All wells screened over entire thickness of aquifer

Steady-State Radial Flow (unlikely)

- well pumped at constant rate
- equilibrium (no change in head with time)

Case 1 - confined aquifer top and bottom

$$Q = (2\pi rb)K\frac{dh}{dr} = 2\pi rT\frac{dh}{dr}$$

where Q = pumping rate
K = hydraulic conductivity
b = aquifer thickness
h = hydraulic head
r = radial distance to well
T = transmissivity

If 2 observation wells, then can integrate and solve for transmissivity

$$\int_{h_1}^{h_2} dh = \frac{Q}{2\pi T}\int_{r_1}^{r_2} \frac{dr}{r}$$

$$h_2 - h_1 = \frac{Q}{2\pi T} \ln\left(\frac{r_2}{r_1}\right)$$

$$T = \frac{Q}{2\pi(h_2 - h_1)} \ln\left(\frac{r_2}{r_1}\right)$$

Case 2 - unconfined aquifer

$$Q = (2\pi rh)K\frac{dh}{dr}$$

$$K = \frac{Q}{\pi(h_2^2 - h_1^2)} \ln\left(\frac{r_2}{r_1}\right)$$

Note - for steady state conditions canNOT determine storativity of aquifer as hydraulic head does not change with time

Nonequilibrium Flow Conditions

Theis Method (Nonequilibrium Radial Flow, Confined Aquifer)

$$T = [Q/(4\pi(h_o-h)]W(u)$$

$$S = \frac{4Ttu}{r^2}$$

where T = transmissivity
Q = steady pumping rate
h_0 - h = drawdown
W = well function
u = dimensionless constant
S = storativity

Procedure

Generate W(u) versus u on log-log paper (from Appendix 1)
Plot Drawdown versus time on log-log paper
Overlay the two plots and match the curves
Select match point and read W(u), u, Drawdown and time
Use these values, plus Q and r from well to solve for T and S