Well Test Analysis

5.2 Basic Assumptions

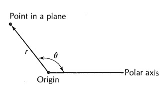
In this chapter we need to make assumptions about the hydraulic conditions in the aquifer and about the pumping and observation wells. In this section we list the basic assumptions that apply to all situations described in the chapter. Each situation will also have additional assumptions.

1. The aquifer is bounded on the bottom by a confining layer.
2. All geologic formations are horizontal and have infinite horizontal extent.
3. The potentiometric surface of the aquifer is horizontal prior to the start of the pumping.
4. The potentiometric surface of the aquifer is not changing with time prior to the start of the pumping.
5. All changes in the position of the potentiometric surface are due to the effect of the pumping well alone.
6. The aquifer is homogeneous and isotropic.
7. All flow is radial toward the well.
8. Ground-water flow is horizontal.
9. Darcy's law is valid.
10. Ground water has a constant density and viscosity.
11. The pumping well and the observation wells are fully penetrating; that is, they are screened over the entire thickness of the aquifer.
12. The pumping well has an infinitesimal diameter and is 100% efficient.

Polar Coordinates

Two-dimensional flow in a confined aquifer has previously been derived as Equation 4.42, which is

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{S}{T}\frac{\partial h}{\partial t}$$



This equation is expressed in Cartesian coordinates, which are based on an x-y grid. In an isotropic and homogeneous aquifer with radial symmetry, we can transform Equation 4.42 with the following relationship which comes from the Pythagorean theorem:

$$r = \sqrt{x^2 + y^2} \quad (5.1)$$

The result is Equation 4.42 in radial coordinates

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{T}\frac{\partial h}{\partial t} \quad (5.2)$$

where

- h is hydraulic head (L; m or ft)
- S is storativity (dimensionless)
- T is transmissivity (L^2/T; m^2/d or ft^2/d)
- t is time [1](T; d)
- r is radial distance from the pumping well (L; m or ft)

One 12" Pizza = Nine 4" Pizzas

Polar Coordinates

The two-dimensional equation for confined flow, if there is recharge to the aquifer, is given by Equation 4.44, which is:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{w}{T} = \frac{S}{T}\frac{\partial h}{\partial t}$$

Equation 4.44 can likewise be transformed into radial coordinates becoming

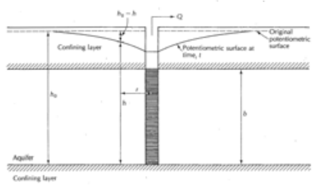
$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} + \frac{w}{T} = \frac{S}{T}\frac{\partial h}{\partial t} \qquad (5.3)$$

where w is the rate of vertical leakage (L/T; m/d or ft/d)

Computing Drawdown from a Pumping Well: Theis

The first mathematical analysis of transient drawdown effects in a confined aquifer was published by C. V. Theis (1935). Theis made the following assumptions in addition to the basic assumptions of Section 5.2.

1. The aquifer is confined top and bottom.
2. There is no source of recharge to the aquifer.
3. The aquifer is compressible and water is released instantaneously from the aquifer as the head is lowered.
4. The well is pumped at a constant rate. Figure 5.2 illustrates the aquifer conditions.



▲ FIGURE 5.2
Fully penetrating well pumping from a confined aquifer.

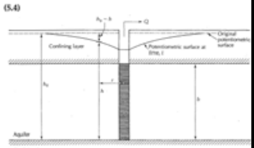
Computing Drawdown from a Pumping Well: Theis

From Darcy's law, the flow of water through any circular section of the aquifer toward the well is the area of the circular section times the hydraulic conductivity, K, times the hydraulic gradient. The hydraulic gradient is expressed as the change in head with radial distance from the well, dh/dr. The area of a circular section is the circumference of the circular section, $2\pi r$, where r is the radius of the circle, times the thickness of the aquifer, b. Therefore, the area is $2\pi rb$. Flow through the circular section can be expressed as

$$Q = (2\pi rb)K\left(\frac{dh}{dr}\right) \qquad (5.4)$$

where

- Q is the pumping rate (L^3/T)
- r is the radial distance from the circular section to the well (L)
- b is the aquifer thickness (L)
- K is the hydraulic conductivity (L/T)
- dh/dr is the hydraulic gradient (dimensionless)



▲ FIGURE 5.2
Fully penetrating well pumping from a confined aquifer.

Since transmissivity (T) is the product of the aquifer thickness and the hydraulic conductivity, Equation 5.4 can also be expressed as

$$Q = 2\pi rT\left(\frac{dh}{dr}\right) \qquad (5.5)$$

Computing Drawdown from a Pumping Well: Theis

Equation 5.5 can be rearranged as follows:

$$r\frac{\partial h}{\partial r} = \frac{Q}{2\pi T} \quad (5.6)$$

The basic assumptions can be expressed mathematically as initial and boundary conditions. The initial condition of a horizontal potentiometric surface is

$$h(r,0) = h_0 \text{ for all } r \quad (5.7)$$

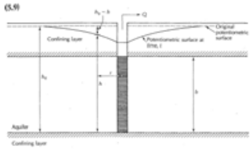
The boundary condition signifying an infinite horizontal extent with no drawdown at any time is

$$h(\infty, t) = h_0 \text{ for all } t \quad (5.8)$$

The constant pumping rate, Q, is given by Equation 5.6.

The solution that Theis arrived at for Equation 5.2 under the initial and boundary conditions of Equations 5.6, 5.7, and 5.8 is known as the **Theis** or **nonequilibrium equation**:*

$$h_0 - h = \frac{Q}{4\pi T}\int_u^\infty \frac{e^{-a}}{a}da \quad (5.9)$$



▲ FIGURE 5.2
Fully penetrating well pumping from a confined aquifer.

Computing Drawdown from a Pumping Well: Theis

where the argument u is given by

$$u = \frac{r^2 S}{4Tt} \quad (5.10)$$

where

- Q is the constant pumping rate (L^3/T; m^3/d or ft^3/d)
- h is the hydraulic head (L; m or ft)
- h_0 is the initial hydraulic head (L; m or ft)
- h_0-h is the drawdown (L; m or ft)
- T is the aquifer transmissivity (L^2/T; m^2/d or ft^2/d)
- t is the time since pumping began (T; d)
- r is the radial distance from the pumping well (L; m or ft)
- S is the aquifer storativity (dimensionless)

It should be noted that Q is a pumping rate in cubic meter or cubic feet per day. Even if the well is pumped for less than 24 hs, the rate of Q must still be expressed in terms of the volume that would be pumped in a day.

The integral in Equation 5.9 is called the exponential integral. It can be approximated by an infinite series so that the Theis equation becomes

$$h_0 - h = \frac{Q}{4\pi T}\left[-0.5772 - \ln u + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} - \frac{u^4}{4 \cdot 4!} + \ldots\right] \quad (5.11)$$

Example: Theis Eqn.

Using well function notation, the Theis equation is also expressed as

$$h_0 - h = \frac{Q}{4\pi T}W(u) \quad (5.12)$$

EXAMPLE PROBLEM

A well is located in an aquifer with a conductivity of 14.9 m/d and a storativity of 0.0051. The aquifer is 20.1 m thick and is pumped at a rate of 2725 m^3/d. What is the drawdown at a distance of 7.0 m from the well after 1 day of pumping?

$$T = Kb = 14.9 \text{ m/d} \times 20.1 \text{ m} = 299 \text{ m}^2/\text{d}$$

$$u = \frac{r^2 S}{4Tt} = \frac{(7.0 \text{ m})^2 \times 0.0051}{4 \times 299 \text{ m}^2/\text{d} \times 1 \text{ d}} = 0.00021$$

From the table of $W(u)$ and u, if $u = 2.0 \times 10^{-4}$, $W(u) = 7.94$:

$$h_0 - h = \frac{Q}{4\pi T}W(u) = \frac{2725 \text{ m}^3/\text{d} \times 7.94}{4 \times \pi \times 299 \text{ m}^2/\text{d}} = 5.7 \text{ m}$$

Leaky, Confined Aquifer

Equation 5.3 is the two-dimensional flow equation if there is vertical leakage into the aquifer. The leakage rate, w, can be found from Darcy's law. The vertical hydraulic conductivity of the leaky confining layer is K'. The hydraulic gradient is the head across the leaky confining layer, which is the drawdown, h_0-h, divided by the thickness of the confining layer, b'. The leakage rate is therefore

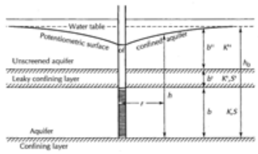
$$w = K'\frac{h_0 - h}{b'} \tag{5.13}$$

By substituting Equation 5.13 into Equation 5.3 we obtain

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} - \frac{(h_0 - h)K'}{Tb'} = \frac{S}{T}\frac{\partial h}{\partial t} \tag{5.14}$$

where

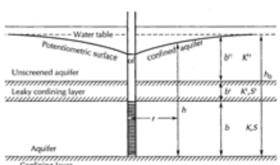
- K' is the vertical hydraulic conductivity of the leaky layer (L/T)
- b' is the thickness of the leaky layer (L)
- h is the head (L)
- r is the radial distance from the pumping well (L)
- t is the time (T)
- S is the storativity (dimensionless)
- T is the transmissivity (L^2/T)
- h_0-h is the drawdown (L)



▲ FIGURE 5.3
Fully penetrating well in an aquifer overlain by a semipermeable confining layer.

No Water Drains From Confining Layer Solution

1. The aquifer is bounded on the top by an aquitard.
2. The aquitard is overlain by an unconfined aquifer, known as the *source bed*.
3. The water table in the source bed is initially horizontal.
4. The water table in the source bed does not fall during pumping of the aquifer.
5. Ground-water flow in the aquitard is vertical.
6. The aquifer is compressible, and water drains instantaneously with a decline in head.
7. The aquitard is incompressible, so that no water is released from storage in the aquitard when the aquifer is pumped.



▲ FIGURE 5.3
Fully penetrating well in an aquifer overlain by a semipermeable confining layer.

Test of Assumption #4

Assumption 4, that the water table does not decline during pumping, is difficult to attain unless there is continuous recharge to the water-table aquifer. However, it can be considered to be valid when either of the following conditions is true (Neuman & Witherspoon 1969):

$$t < \frac{S'(b')^2}{10bK'} \tag{5.15A}$$

or

$$b''K'' > 100bK \tag{5.15B}$$

where

- t is time since pumping began (T; d)
- S' is storativity of aquitard (dimensionless)
- b' is thickness of aquitard (L; ft or m)
- b is thickness of confined aquifer (L; ft or m)
- b'' is saturated thickness of water-table aquifer (L; ft or m)
- K' is vertical hydraulic conductivity of aquitard (L/T; ft/d or m/d)
- K'' is hydraulic conductivity of water-table aquifer (L/T; ft/d or m/d)
- K is hydraulic conductivity of confined aquifer (L/T; ft/d or m/d)

Test of Assumption #7

Assumption 7, that no water is released from the aquitard, is valid under either of two conditions. Hantush (1960b) showed that the effects of water released from the aquitard are negligible if

$$t > 0.036b'S'/K' \quad \textbf{(5.16)}$$

Neuman and Witherspoon (1969) also showed that the assumption is valid when

$$r < 0.04b[(KS_s/K'S'_s)]^{1/2} \quad \textbf{(5.17)}$$

where

S_s is the specific storage of the confined aquifer ($1/L$; 1/ft or 1/m)

S'_s is the specific storage of the aquitard ($1/L$; 1/ft or 1/m)

Although the basic assumptions of Section 5.2 include an infinitesimal well diameter, the following solution is valid for any well diameter, provided that

$$t > (30r_w^2S/T)\,[1 - (10r_w/b)^2] \quad \textbf{(5.18)}$$

and

$$r_w/(Tb'/K')^{1/2} < 0.1$$

where

r_w is the radius of the pumping well (L; ft or m)

S is storativity of the confined aquifer (dimensionless)

T is transmissivity of the confined aquifer (L^2/T; ft^2/d or m^2/d)

Hantush-Jacob Formula

The solution to Equation 5.14 as given by Hantush (1956, 1960b) and Hantush and Jacob (1954) is known as the **Hantush-Jacob formula** and is

$$h_0 - h = \frac{Q}{4\pi T} W(u, r/B) \quad \textbf{(5.20)}$$

$$u = \frac{r^2S}{4Tt} \quad \textbf{(5.21)}$$

$$B = (Tb'/K')^{1/2} \quad \textbf{(5.22)}$$

where

Q	is the pumping rate (L^3/T; ft^3/d or m^3/d)
$h_0 - h$	is the drawdown in the confined aquifer (L; ft or m)
T	is the transmissivity of the confined aquifer (L^2/T; ft^2/d or m^2/d)
$W(u, r/B)$	is the leaky artesian well function (Values of this function are tabulated in Appendix 3.)
r	is the distance from the pumping well to the observation well (L; ft or m)
S	is the storativity of the confined aquifer (dimensionless)
t	is time since pumping began (T; d)
B	is leakage factor (L; ft or m)
b'	is thickness of the aquitard (L; ft or m)
K'	is hydraulic conductivity of the aquitard (L/T; ft/d or m/d)

The rate that water is being drawn from elastic storage in the confined aquifer, q_s (L^3/T; ft^3/d or m^3/d), at the specific time, t (T; d), since pumping began may be determined from

$$q_s = Q\exp(-Tt/SB^2) \quad \textbf{(5.23)}$$

If the total discharge at time t is Q and the water drawn from storage at that time is q_s, then the rate that water is coming from leakage across the aquitard at that time, q_L, is found from

$$q_L = Q - q_s \quad \textbf{(5.24)}$$

Leaky, Confined Aquifer Example 1

A confined aquifer is underlain by an aquiclude and overlain by an aquitard and a water-table aquifer. The following aquifer characteristics are given.

Confined aquifer: $b = 5.2$ m, $K = 0.73$ m/d, $S = 0.0035$, $T = 3.8$ m^2/d

Aquitard: $b' = 1.1$ m, $K' = 5.5 \times 10^{-5}$ m/d, $S' = 0.00061$

Water-table aquifer: $b'' = 25$ m, $K'' = 35$ m/d

A well that fully penetrates the aquifer has a radius of 0.15 m. If it is pumped at a rate of 28 m^3/d, what is the drawdown after 1 d of pumping at the following distance from the well: 1.5 m, 5.5 m, 10 m, 25 m, 75 m, 150 m?

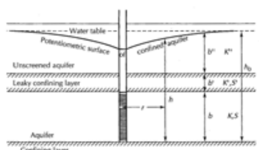
First, test to see if the assumptions for a leaky, confined aquifer with negligible storage in the aquitard are valid. To test to see if there will be negligible decline in the water level in the water-table aquifer, use Equation 5.15B.

$$b''K'' > 100bK$$

$$25 \text{ m/d} \times 35 \text{ m} > 100 \times 5.2 \text{ m} \times 0.73 \text{ m/d}$$

$$875 \text{ m}^2/\text{d} > 380 \text{ m}^2/\text{d}$$

Therefore, the assumption is valid.



▲ FIGURE 5.3
Fully penetrating well in an aquifer overlain by a semipermeable confining layer.

Leaky, Confined Aquifer Example 1

To test if the assumption that the contribution from storage in the aquitard is negligible, use Equation 5.16.

$$t > 0.036 b'S'/K'$$
$$1\ \text{d} > (0.036 \times 1.1\ \text{m} \times 0.00061)/(5.5 \times 10^{-5}\ \text{m/d})$$
$$1\ \text{d} > 0.44\ \text{d}$$

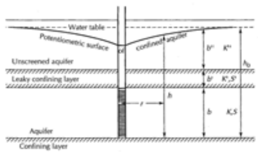
Therefore, the assumption is valid.

A confined aquifer is underlain by an aquiclude and overlain by an aquitard and a water-table aquifer. The following aquifer characteristics are given.

Confined aquifer: $b = 5.2$ m, $K = 0.73$ m/d, $S = 0.0035$, $T = 3.8$ m^2/d

Aquitard: $b' = 1.1$ m, $K' = 5.5 \times 10^{-5}$ m/d, $S' = 0.00061$

Water-table aquifer: $b'' = 25$ m, $K'' = 35$ m/d



▲ FIGURE 5.3
Fully penetrating well in an aquifer overlain by a semipermeable confining layer.

Leaky, Confined Aquifer Example 1

To test if the radius of the well can be considered negligible, use Equations 5.18 and 5.19.

$$t > (30 r_w^2 S/T)[1 - (10 r_w/b)^2]$$
$$1\ \text{d} > [30 \times (0.15\ \text{m})^2 \times (0.0035/3.8\ \text{m}^2/\text{d})][1 - (10 \times 0.15\ \text{m}/5.2\ \text{m})^2]$$
$$1\ \text{d} > (6.2 \times 10^{-4}\ \text{d})(1 - 0.08)$$
$$1\ \text{d} > 5.7 \times 10^{-4}\ \text{d}$$

$$r_w/(Tb'/K')^{1/2} < 0.1$$
$$0.15\ \text{m}/[(3.8\ \text{m}^2/\text{d} \times 1.1\ \text{m})/(5.5 \times 10^{-5}\ \text{m/d})]^{1/2} < 0.1$$
$$(0.15\ \text{m})/(275\ \text{m}) < 0.1$$
$$5.5 \times 10^{-4} < 0.1$$

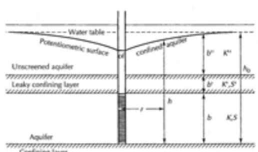
Both the preceding conditions are met, so this assumption is also true and the Hantush-Jacob equations can be used.

A confined aquifer is underlain by an aquiclude and overlain by an aquitard and a water-table aquifer. The following aquifer characteristics are given.

Confined aquifer: $b = 5.2$ m, $K = 0.73$ m/d, $S = 0.0035$, T

Aquitard: $b' = 1.1$ m, $K' = 5.5 \times 10^{-5}$ m/d, $S' = 0.00061$

Water-table aquifer: $b'' = 25$ m, $K'' = 35$ m/d



▲ FIGURE 5.3
Fully penetrating well in an aquifer overlain by a semipermeable confining layer.

Leaky, Confined Aquifer Example 1

To find $W(u, r/B)$, we must first find u and r/B. As we wish to know these parameters for several values of r, we will first find them with respect to r. From Equation 5.21,

$$u = (r^2 S)/(4Tt)$$
$$= [(r^2\text{m}^2) \times 0.0035]/(4 \times 3.8\ \text{m}^2/\text{d} \times 1\ \text{d})$$
$$= (2.3 \times 10^{-4})r^2$$
$$r/B = r/(Tb'/K')^{1/2}$$
$$= r\text{m}/[(3.8\ \text{m}^2/\text{d} \times 1.1\ \text{m})/(5.5 \times 10^{-5}\ \text{m/d})]^{1/2}$$
$$= r/275$$

Confined aquifer: $b = 5.2$ m, $K = 0.73$ m/d, $S = 0.0035$, $T = 3.8$ m^2/d

Aquitard: $b' = 1.1$ m, $K' = 5.5 \times 10^{-5}$ m/d, $S' = 0.00061$

Water-table aquifer: $b'' = 25$ m, $K'' = 35$ m/d

Once values of u and r/B are determined for each value of r, then the value of $W(u, r/B)$ is found in Appendix 3. The drawdown is then found from Equation 5.20.

$$h_0 - h = \frac{Q}{4\pi T} W(u, r/B)$$
$$= 28\ \text{m}^3/\text{d}/(4\pi \times 3.8\ \text{m}^2/\text{d}) W(u, r/B)$$
$$= 0.59 W(u, r/B)\ \text{m}$$

r	u	r/B	$W(u, r/B)$	$h_0 - h$
1.5 m	5.17×10^{-4}	5.45×10^{-3}	7.0	4.1 m
5.5 m	6.96×10^{-3}	2.00×10^{-2}	4.4	2.6 m
10 m	2.30×10^{-2}	3.64×10^{-2}	3.2	1.9 m
25 m	1.44×10^{-1}	9.09×10^{-2}	1.5	0.89 m
75 m	1.29	2.73×10^{-1}	0.17	0.10 m
150 m	5.18	5.45×10^{-1}	0.0017	0.0010 m

Leaky, Confined Aquifer Example 1

If the well is pumped long enough, all the water will be coming from leakage across the confining layer and none from elastic storage in the confined aquifer (Hantush & Jacob 1954). This occurs when

$$t > \frac{8b'S}{K'} \qquad (5.25)$$

In this case the drawdown can be found from (Hantush & Jacob 1954:):

$$h_0 - h = \frac{Q}{2\pi T} K_0(r/B) \qquad (5.26)$$

where K_0 is the zero-order modified Bessel function of the second kind. A partial listing of Bessel functions is found in Appendix 5.

Some Water Drains From Confining Layer Solution

This case has two solutions. During the early part of pumping, when the following condition is met, all the water will come from elastic storage in the aquifer and the aquitard. The early-time condition is

$$t < b'S'/10K' \qquad (5.27)$$

The solution to Equation 5.14 is

$$h_0 - h = \frac{Q}{4\pi T} H(u, \beta) \qquad (5.28)$$

where $H(u, \beta)$ is a function with values tabulated in Appendix 4 and

$$\beta = \frac{r}{4B}(S'/S)^{1/2} \qquad (5.29)$$

$$B = \left(\frac{Tb'}{K'}\right)^{1/2} \qquad (5.30)$$

$$u = \frac{r^2 S}{4Tt} \qquad (5.31)$$

The rate of flow from storage in the main aquifer, q_s, is given by

$$q_s = Q \exp(\nu t) \operatorname{erfc}(\sqrt{\nu t}) \qquad (5.32)$$

where

$$\nu = (K'/b')(S'/S^2)\ (1/T;\ 1/d) \qquad (5.33)$$

Some Water Drains From Confining Layer Solution

If sufficient time elapses, the aquifer will reach equilibrium and all of the water will be coming from drainage from the overlying source bed. The time to reach this equilibrium is

$$t > \frac{8[S + (S'/3) + S'']}{[(K'/b') + (K''/b'')]^{1/2}} \qquad (5.34)$$

If the value of r_w/B is less than 0.01, then the solution is

$$h_0 - h = \frac{Q}{2\pi T} K_0(r/B) \qquad (5.35)$$

This is the same solution as that for equilibrium conditions in the case where no water comes from elastic storage in the aquitard, since all the water comes from the source bed.

Leaky, Confined Aquifer Example 2

A confined aquifer is overlain by an aquitard and a water-table aquifer. The layers have the following characteristics.

Confined aquifer: $b = 4.3$ m, $K = 1.1$ m/d, $S = 0.00053$, $T = 4.7$ m^2/d
Aquitard: $b' = 7.2$ m, $K' = 5.5 \times 10^{-6}$ m/d, $S' = 0.00012$
Source bed: $b'' = 17$ m, $K'' = 87$ m/d, $S'' = 0.055$

If a well is pumped at a rate of 15 m^3/d for 1.76 d, what would the drawdown be at a distance of 22 m?

A confined aquifer is underlain by an aquiclude and overlain by an aquitard aquifer. The following aquifer characteristics are given.
Confined aquifer: $b = 5.2$ m, $K = 0.73$ m/d, $S = 0.0035$, $T = 3.8$ m^2/d
Aquitard: $b' = 1.1$ m, $K' = 5.5 \times 10^{-[illegible]}$ m/d, $S' = 0.00061$
Water-table aquifer: $b'' = 25$ m, $K'' = 35$ m/d

First we need to test if the assumption that the head in the source bed will remain constant is valid. This is done with Equation 5.15B:

$$b''K'' > 100bK$$
$$17 \text{ m} \times 87 \text{ m/d} > 100 \times 4.3 \text{ m} \times 1.1 \text{ m/d}$$
$$1497 \text{ m}^2\text{/d} > 473 \text{ m}^2\text{/d}$$

Therefore, this assumption is true.

We then use Equation 5.16 to see if the contribution of water from elastic storage in the aquifer needs to be considered.

$$t > 0.036b'S'/K'$$
$$1.76 \text{ d} > (0.036 \times 7.2 \text{ m} \times 0.00012)/(5.5 \times 10^{-6} \text{ m/d})$$
$$1.76 \text{ d} \ngtr 5.66 \text{ d}$$

The conditional statement is not true; therefore we must consider the effects of storage

Leaky, Confined Aquifer Example 2

A confined aquifer is overlain by an aquitard and a water-table aquifer. The layers have the following characteristics.

Confined aquifer: $b = 4.3$ m, $K = 1.1$ m/d, $S = 0.00053$, $T = 4.7$ m^2/d
Aquitard: $b' = 7.2$ m, $K' = 5.5 \times 10^{-6}$ m/d, $S' = 0.00012$
Source bed: $b'' = 17$ m, $K'' = 87$ m/d, $S'' = 0.055$

If a well is pumped at a rate of 15 m^3/d for 1.76 d, what would the drawdown be at a distance of 22 m?

We need to know if we should use the early-time equation, where water is coming from the elastic storage in the aquitard, or the equilibrium equation for confined aquifers. We will start with the early-time test, Equation 5.27.

$$t < b'S'/10K'$$
$$1.76 \text{ d} < 7.2 \text{ m} \times 0.00012 \;/\; 10 \times 5.5 \times 10^{-6} \text{ m/d}$$
$$1.76 \text{ d} < 15.7 \text{ d}$$

The conditional statement is true; therefore we must use the early-time equation, 5.28.

$$h_0 - h = \frac{Q}{4\pi T} H(u, \beta)$$

Leaky, Confined Aquifer Example 2

A confined aquifer is overlain by an aquitard and a water-table aquifer. The layers have the following characteristics.

Confined aquifer: $b = 4.3$ m, $K = 1.1$ m/d, $S = 0.00053$, $T = 4.7$ m^2/d
Aquitard: $b' = 7.2$ m, $K' = 5.5 \times 10^{-6}$ m/d, $S' = 0.00012$
Source bed: $b'' = 17$ m, $K'' = 87$ m/d, $S'' = 0.055$

If a well is pumped at a rate of 15 m^3/d for 1.76 d, what would the drawdown be at a distance of 22 m?

The first step is to find the value of $H(u, \beta)$:

$$u = \frac{r^2 S}{4Tt}$$
$$= [(22 \text{ m})^2 \times 0.00053]/[4 \times 4.7 \text{ m}^2\text{/d} \times 1.76 \text{ d}]$$
$$= 7.75 \times 10^{-3}$$
$$\beta = \frac{r}{4B}(S'/S)^{1/2}$$
$$B = \left(\frac{Tb'}{K'}\right)^{1/2}$$
$$= [4.7 \text{ m}^2\text{/d} \times 7.2 \text{ m} \;/\; 5.5 \times 10^{-6} \text{ m/d}]^{1/2}$$
$$= 2.5 \times 10^3$$

Leaky, Confined Aquifer Example 2

A confined aquifer is overlain by an aquitard and a water-table aquifer. The layers have the following characteristics.

Confined aquifer: $b = 4.3$ m, $K = 1.1$ m/d, $S = 0.00053$, $T = 4.7$ m^2/d
Aquitard: $b' = 7.2$ m, $K' = 5.5 \times 10^{-6}$ m/d, $S' = 0.00012$
Source bed: $b'' = 17$ m, $K'' = 87$ m/d, $S'' = 0.055$

If a well is pumped at a rate of 15 m^3/d for 1.76 d, what would the drawdown be at a distance of 22 m?

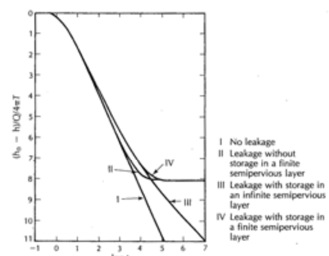
$$\beta = \frac{r}{4B}(S'/S)^{1/2}$$

$$\beta = (22\text{ m} / 4 \times 2.48 \times 10^3) \times (0.00012/0.00053)^{1/2}$$
$$= 2.22 \times 10^{-3} \times 0.476$$
$$= 1.1 \times 10^{-3}$$

With the values of u and β as calculated, the value of $H(u, \beta)$ found from Appendix 4 is 4.3. This is substituted into Equation 5.28.

$$h_0 - h = \frac{Q}{4\pi T}H(u, \beta)$$
$$= [15.3\text{ m}^3/\text{d}/(4 \times 3.1416 \times 4.7\text{ m}^3/\text{d})] \times 4.3$$
$$= 1.1\text{ m}$$

Summary Drawdown Responses for Confined Aquifers



▲ FIGURE 5.4
Plots of log of dimensionless drawdown as a function of log time for an aquifer with various types of overlying confining layer. *Source: M. S. Hantush, Journal of Geophysical Research 65 (1960): 3713–25. Used with permission.*

Unconfined Aquifers

5.4.3 Flow in an Unconfined Aquifer

The flow of water in an unconfined aquifer toward a pumping well is described by the following equation (Neuman & Witherspoon 1969):

$$K_r \frac{\partial^2 h}{\partial r^2} + \frac{K_r}{r}\frac{\partial h}{\partial r} + K_v \frac{\partial^2 h}{\partial z^2} = S_s \frac{\partial h}{\partial t} \qquad (5.36)$$

where

- h is the saturated thickness of the aquifer (L; ft or m)
- r is radial distance from the pumping well (L; ft or m)
- z is elevation above the base of the aquifer (L; ft or m)
- S_s is specific storage ($1/L$; 1/ft or 1/m)
- K_r is radial hydraulic conductivity (L/T; ft/d or m/d)
- K_v is vertical hydraulic conductivity (L/T; ft/day or m/d)
- t is time (T; d)

Unconfined Aquifers: Assumptions

1. The aquifer is unconfined.
2. The vadose zone has no influence on the drawdown.
3. Water initially pumped comes from the instantaneous release of water from elastic storage.
4. Eventually water comes from storage due to gravity drainage of interconnected pores.
5. The drawdown is negligible compared with the saturated aquifer thickness.
6. The specific yield is at least 10 times the elastic storativity.
7. The aquifer may be—but does not have to be—anisotropic with the radial hydraulic conductivity different than the vertical hydraulic conductivity.

Unconfined Aquifers: Neuman's Solution

$$h_0 - h = \frac{Q}{4\pi T} W(u_A, u_B, \Gamma) \qquad (5.37)$$

where $(W(u_A, u_B, \Gamma)$ is the well function for the water-table aquifer, as tabulated in Appendix 6.

$$u_A = \frac{r^2 S}{4Tt} \text{ (for early drawdown data)} \qquad (5.38)$$

$$u_B = \frac{r^2 S_y}{4Tt} \text{ (for later drawdown data)} \qquad (5.39)$$

$$\Gamma = \frac{r^2 K_v}{b^2 K_h} \qquad (5.40)$$

where

- $h_0 - h$ is the drawdown (L; ft or m)
- Q is the pumping rate (L^3/T; ft^3/d or m^3/d)
- T is the transmissivity (L^2/T; ft^2/d or m^2/d)
- r is the radial distance from the pumping well (L; ft or m)
- S is the storativity (dimensionless)
- S_y is the specific yield (dimensionless)
- t is the time (T; d)
- K_h is the horizontal hydraulic conductivity (L/T; ft/d or m/d)
- K_v is the vertical hydraulic conductivity (L/T; ft/d or m/d)
- b is the initial saturated thickness of the aquifer (L; ft or m)

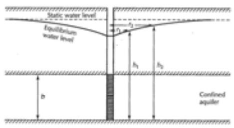
Aquifer Parameters from Time-Drawdown Data

General Assumptions

1. The pumping well is screened only in the aquifer being tested.
2. All observation wells are screened only in the aquifer being tested.
3. The pumping well and the observation wells are screened throughout the entire thickness of the aquifer.

Confined Aquifer Parameters from Time-Drawdown Data

Steady Radial Flow Assumptions



1. The aquifer is confined top and bottom.
2. The well is pumped at a constant rate.
3. Equilibrium has been reached; that is, there is no further change in drawdown with time.

Confined Aquifer Parameters from Time-Drawdown Data: Thiem Eqn.

We have previously derived Equation 5.5, which uses Darcy's law to yield the discharge of a well in a situation with horizontal, confined flow. Equation 5.5 can be rearranged to provide the following

$$dh = \frac{Q}{2\pi T}\frac{dr}{r} \tag{5.41}$$

If there are two observation wells, the head is h_1 at a distance r_1 from the pumping well; it is h_2 at a distance r_2. We can integrate both sides of Equation 5.41 with these boundary conditions

$$\int_{h_1}^{h_2} dh = \frac{Q}{2\pi T}\int_{r_1}^{r_2}\frac{dr}{r} \tag{5.42}$$

This can be solved to yield

$$h_2 - h_1 = \frac{Q}{2\pi T}\ln\left(\frac{r_2}{r_1}\right) \tag{5.43}$$

Equation 5.43 can be rearranged to yield the Thiem equation for confined aquifers:

$$\boxed{T = \frac{Q}{2\pi(h_2 - h_1)}\ln\left(\frac{r_2}{r_1}\right)} \tag{5.44}$$

where

- T is aquifer transmissivity (L^2/T; ft^2/d or m^2/d)
- Q is pumping rate (L^3/T; ft^3/d or m^3/d)
- h_1 is head at distance r_1 from the pumping well (L; ft or m)
- h_2 is head at distance r_2 from the pumping well (L; ft or m)

Confined Aquifer Parameters from Time-Drawdown Data: Thiem Eqn.

A well in a confined aquifer is pumped at a rate of 220 gal/min. Measurement of drawdown in two observation wells shows that after 1270 min of pumping, no further drawdown is occurring. Well 1 is 26 ft from the pumping well and has a head of 29.34 ft above the top of the aquifer. Well 2 is 73 ft from the pumping well and has a head of 32.56 ft above the top of the aquifer. Use the Thiem equation to find the aquifer transmissivity.

We must first convert the pumping rate of 220 gal/min to an equivalent rate in cubic feet per day. We make this conversion, even though steady-state conditions were reached, before a full day (1440 min) of pumping occurred.

$$220\,\frac{\text{gal}}{\text{min}} \times \frac{1\ \text{ft}^3}{7.48\ \text{gal}} \times 1440\,\frac{\text{min}}{\text{d}} = 42{,}400\ \text{ft}^3/\text{d}$$

Now we substitute the given values into Equation 5.44.

$$\begin{aligned} T &= \frac{Q}{2\pi(h_2 - h_1)}\ln\left(\frac{r_2}{r_1}\right) \\ &= \frac{42{,}400\ \text{ft}^3/\text{d}}{2\pi(32.56\ \text{ft} - 29.34\ \text{ft})}\ln\left(\frac{73\ \text{ft}}{26\ \text{ft}}\right) \\ &= \frac{42{,}400}{20.2}\ln(2.81)\ \text{ft}^2/\text{d} \\ &= 2170\ \text{ft}^2/\text{d} \end{aligned}$$

What about Storativity?????

Unconfined Aquifer Parameters from Time-Drawdown Data

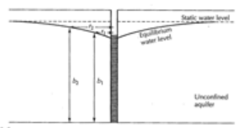
1. The aquifer is unconfined and underlain by a horizontal aquiclude.
2. The well is pumped at a constant rate.
3. Equilibrium has been reached; that is, there is no further change in drawdown with time.

Radial flow in the unconfined aquifer is described by

$$Q = (2\pi rb)K\left(\frac{db}{dr}\right) \quad (5.45)$$

where

- Q is the pumping rate (L^3/T)
- r is the radial distance from the circular section to the well (L)
- b is the saturated thickness of the aquifer (L)
- K is the hydraulic conductivity (L/T)
- db/dr is the hydraulic gradient (dimensionless)



▲ FIGURE 5.5
Equilibrium drawdown: A. confined aquifer; B. unconfined aquifer.

Unconfined Aquifer Parameters from Time-Drawdown Data: Thiem Eqn.

Equation 5.45 can be rearranged as follows:

$$b\,db = \frac{Q}{2\pi K}\frac{dr}{r} \quad (5.46)$$

If there are two observation wells, the head is h_1 at a distance r_1 from the pumping well and it is h_2 at a distance r_2. We can integrate both sides of Equation 5.46 with these boundary conditions:

$$\int_{h_1}^{h_2} b\,db = \frac{Q}{2\pi K}\int_{r_1}^{r_2}\frac{dr}{r} \quad (5.47)$$

This can be solved to yield

$$b_2^2 - b_1^2 = \frac{Q}{\pi K}\ln\left(\frac{r_2}{r_1}\right) \quad (5.48)$$

Equation 5.48 can be rearranged to yield the Thiem equation for an unconfined aquifer:

$$\boxed{K = \frac{Q}{\pi(b_2^2 - b_1^2)}\ln\left(\frac{r_2}{r_1}\right)} \quad (5.49)$$

where

- K is hydraulic conductivity (L/T; ft/d or m/d)
- Q is pumping rate (L^3/T; ft^3/d or m^3/d)
- b_1 is saturated thickness at distance r_1 from the pumping well (L; ft or m)
- b_2 is saturated thickness at distance r_2 from the pumping well (L; ft or m)

Non-equilibrium Flow Conditions: Radial Flow Confined Aquifer (Theis Method)

Equation 5.12 can be rearranged as follows:

$$T = \frac{Q}{4\pi(h_0 - h)}W(u) \quad (5.50)$$

where

- T is the aquifer transmissivity (L^2/T; ft^2/d or m^2/d)
- Q is the steady pumping rate (L^3/T; ft^3/d or m^3/d)
- h_0-h is the drawdown (L; ft or m)
- $W(u)$ is the well function of u (dimensionless)

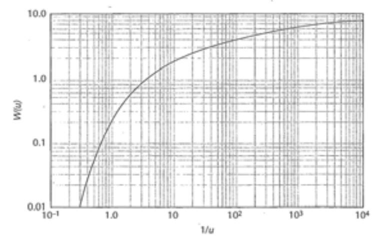
Also, Equation 5.10 can be rearranged as:

$$S = \frac{4Tut}{r^2} \quad (5.51)$$

where

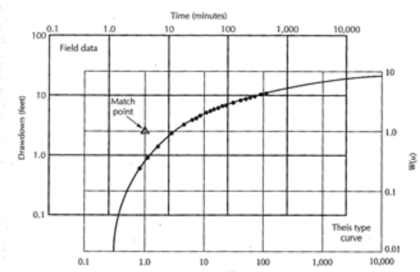
- T is aquifer transmissivity (L^2/T; ft^2/d or m^2/d)
- S is aquifer storativity (dimensionless)
- t is time since pumping began (T; d)
- r is radial distance from the pumping well (L; ft or m)
- u is a dimensionless constant

Theis Curve



▲ FIGURE 5.6
The reverse nonequilibrium type curve (Theis curve) for a fully confined aquifer.

Curve Matching



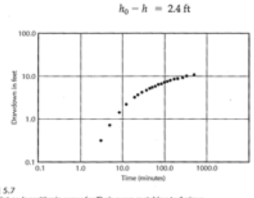
▲ FIGURE 5.8
Match of field-data plot to Theis type curve.

Curve Matching Example

A well in a confined aquifer was pumped at a rate of 220 gal/min for 500 min. The aquifer is 48 ft thick. Time-drawdown data from an observation well located 824 ft away are given in Table 5.1. Find *T*, *K*, and *S*.

The field data are plotted on logarithmic paper (Figure 5.7). The field-data graph is then placed upon the type curve graph (Figure 5.8). The following match point is obtained:

$$W(u) = 1$$
$$1/u = 1$$
$$h_0 - h = 2.4 \text{ ft}$$



▲ FIGURE 5.7
Field-data plot on logarithmic paper for Theis curve-matching technique.

Table 5.1

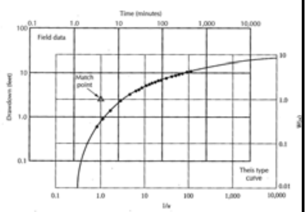
Time after pumping started (min)	Drawdown (ft)
3	0.3
5	0.7
8	1.3
12	2.1
20	3.2
24	3.6
30	4.1
38	4.7
47	5.1
50	5.3
60	5.7
70	6.1
80	6.3
90	6.7
100	7.0
130	7.5
160	8.3
200	8.5
260	9.2
320	9.7
380	10.2
500	10.9

Curve Matching Example

A well in a confined aquifer was pumped at a rate of 220 gal/min for 500 min. The aquifer is 48 ft thick. Time-drawdown data from an observation well located 824 ft away are given in Table 5.1. Find T, K, and S.

The field data are plotted on logarithmic paper (Figure 5.7). The field-data graph is then placed upon the type curve graph (Figure 5.8). The following match point is obtained:

$$W(u) = 1$$
$$1/u = 1$$
$$h_0 - h = 2.4 \text{ ft}$$
$$t = 4.1$$



▲ FIGURE 5.8
Match of field-data plot to Theis type curve.

Curve Matching Example

A well in a confined aquifer was pumped at a rate of 220 gal/min for 500 min. The aquifer is 48 ft thick. Time-drawdown data from an observation well located 824 ft away are given in Table 5.1. Find T, K, and S.

The field data are plotted on logarithmic paper (Figure 5.7). The field-data graph is then placed upon the type curve graph (Figure 5.8). The following match point is obtained:

$$W(u) = 1$$
$$1/u = 1$$
$$h_0 - h = 2.4 \text{ ft}$$
$$t = 4.1 \text{ min}$$

First, time must be converted to days.

$$t = 4.1 \text{ min} \times 1/1440 \text{ d/min} = 2.9 \times 10^{-3} \text{ d}$$

Next the pumping rate of 220 gal/min must be converted to ft^3/d.

$$220 \text{ gal/min} \times 1/7.48 \text{ ft}^3/\text{gal} \times 1440 \text{ min/d} = 42{,}400 \text{ ft}^3/\text{d}$$

Transmissivity is found from Equation 5.50.

$$T = \frac{Q}{4\pi(h_0 - h)} W(u)$$
$$= \frac{42{,}400 \text{ ft}^3/\text{d}}{4 \times \pi \times 2.4 \text{ ft}}$$
$$= 1400 \text{ ft}^2/\text{d}$$

Hydraulic conductivity is transmissivity divided by aquifer thickness.

$$K = \frac{T}{b}$$
$$= \frac{1400 \text{ ft}^2/\text{d}}{48 \text{ ft}}$$
$$= 29 \text{ ft/d}$$

Storativity is found from Equation 5.51.

$$S = \frac{4Tut}{r^2}$$
$$= \frac{4 \times 1400 \text{ ft}^2/\text{d} \times 1 \times 2.9 \times 10^{-3} \text{ d}}{824 \text{ ft} \times 824 \text{ ft}}$$
$$= 2.4 \times 10^{-5}$$

Cooper-Jacob Method

$$T = \frac{Q}{4\pi(h_0 - h)}\left[-0.5772 - \ln\left(\frac{r^2S}{4Tt}\right)\right] \quad (5.52)$$

or

$$T = \frac{Q}{4\pi(h_0 - h)}\left[-\ln(1.78) - \ln\left(\frac{r^2S}{4Tt}\right)\right] \quad (5.53)$$

Combining natural logs we obtain

$$T = \frac{Q}{4\pi(h_0 - h)} \ln\left(\frac{4Tt}{1.78r^2S}\right) \quad (5.54)$$

We can convert this to base 10 logs and simplify:

$$T = \frac{2.3Q}{4\pi(h_0 - h)} \log\left(\frac{2.25Tt}{r^2S}\right) \quad (5.55)$$

$$T = \frac{2.3Q}{4\pi\Delta(h_0 - h)} \quad (5.56)$$

and

$$S = \frac{2.25Tt_0}{r^2} \quad (5.57)$$

where

- T is the transmissivity (L^2/T; ft^2/d or m^2/d)
- Q is the pumping rate (L^3/T; ft^3/d or m^3/d)
- $\Delta(h_0-h)$ is the drawdown per log cycle of time (L; ft or m)
- S is storativity (dimensionless)
- r is the radial distance to the well (L; ft or m)
- t_0 is the time, where the straight line intersects the zero-drawdown axis (T; d)

Cooper-Jacob Method Example

Evaluate the pumping test data of Table 5.1 by the Cooper-Jacob method.

The field data are plotted on semilogarithmic paper (Figure 5.9). A straight line is fit to the later time data and extended back to the zero-drawdown axis. The value of t_0 is 5.2 min, and the drawdown per log cycle of time is 5.5 ft.

First, t_0 must be converted to d.

$$5.2 \text{ min} \times 1/1440 \text{ d/min} = 3.6 \times 10^{-3} \text{ d}$$

The pumping rate has already been determined to be 42,400 ft^3/day. Substituting into Equation 5.56 gives

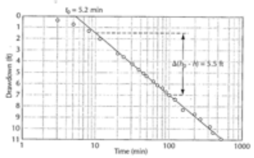
$$T = \frac{2.3Q}{4\pi\Delta(h_0 - h)} = \frac{2.3 \times 42{,}400 \text{ ft}^3/\text{d}}{4 \times \pi \times 5.5 \text{ ft}} = 1400 \text{ ft}^2/\text{d}$$

The value of T can be substituted into Equation 5.57 to find S:

$$S = \frac{2.25Tt_0}{r^2} = \frac{2.25 \times 1400 \text{ ft}^2/\text{d} \times 3.6 \times 10^{-3} \text{ d}}{824 \text{ ft} \times 824 \text{ ft}} = 1.7 \times 10^{-5}$$

Table 5.1

Time after pumping started (min)	Drawdown (ft)
3	0.3
5	0.7
8	1.3
12	2.1
20	3.2
24	3.6
30	4.1
38	4.7
47	5.1
50	5.3
60	5.7
70	6.1
80	6.3
90	6.7
100	7.0
130	7.5
160	8.3
200	8.5
260	9.2
320	9.7
380	10.2
500	10.9



▲ FIGURE 5.9
Cooper-Jacob method of solution of pumping-test data for a fully confined aquifer. Drawdown is plotted as a function of time on semilogarithmic paper.

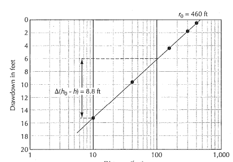
Distance Jacob Method

$$T = \frac{2.3Q}{2\pi\Delta(h_0 - h)} \quad (5.58)$$

$$S = \frac{2.25Tt}{r_0^2} \quad (5.59)$$

where

- T is transmissivity (L^2/T; ft^2/d or m^2/d)
- Q is pumping rate (L^3/T; ft^3/d or m^3/d)
- $\Delta(h_0 - h)$ is drawdown per log cycle of distance (L; ft or m)
- S is storativity (dimensionless)
- t is time (T; d)
- r_0 is the distance at which the straight line intercepts the zero-drawdown axis (L; ft or m)



▲ FIGURE 5.10
Variation of the Jacob method of solution of pumping-test data for a fully confined aquifer. Drawdown is plotted as a function of distance to observation well on semilogarithmic paper.

Distance Jacob Method Example

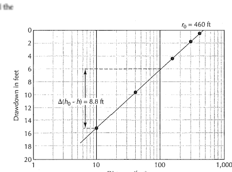
A well pumping at 77,000 ft^3/d has observation wells located 10, 40, 150, 300, and 400 ft away. After 0.14 day of pumping, the following drawdowns were observed:

Distance (ft)	Drawdown (ft)
10	15.1
40	9.4
150	4.4
300	1.7
400	0.25

The data are plotted in Figure 5.10. The drawdown per log cycle is 8.8 ft and r_0 is 460 ft. Find the values of T and S.

$$T = \frac{2.3Q}{2\pi\Delta(h_0 - h)} = \frac{2.3 \times 77{,}000 \text{ ft}^3/\text{d}}{2 \times \pi \times 8.8 \text{ ft}} = 3200 \text{ ft}^2/\text{d}$$

$$S = \frac{2.25Tt}{r_0^2} = \frac{2.25 \times 3200 \text{ ft}^2/\text{d} \times 0.14 \text{ d}}{460 \text{ ft} \times 460 \text{ ft}} = 0.0048$$



▲ FIGURE 5.10
Variation of the Jacob method of solution of pumping-test data for a fully confined aquifer. Drawdown is plotted as a function of distance to observation well on semilogarithmic paper.

Leaky Aquifer, No Storage

$$T = \frac{Q}{4\pi(h_0 - h)} W(u, r/B) \quad (5.60)$$

$$S = \frac{4Tut}{r^2} \quad (5.61)$$

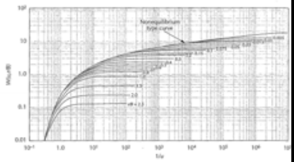
$$r/B = r/(Tb'/K')^{1/2} \quad (5.62)$$

$$K' = [Tb'(r/B)^2]/r^2 \quad (5.63)$$

where

- Q is the pumping rate (L^3/T; ft^3/d or m^3/d)
- T is the transmissivity of the confined aquifer (L^2/T; ft^2/d or m^2/d)
- t is the time since pumping began (T; d)
- h_0–h is the drawdown (L; ft or m)

- S is the storativity of the confined aquifer (dimensionless)
- r is the distance from the pumping well to the observation well (L; ft or m)
- K' is the vertical hydraulic conductivity of the aquitard (L/T; ft/d or m/d)
- b' is the thickness of the aquitard (L; ft or m)
- B is the leakage factor, $(Tb'/K')^{1/2}$ (L; ft or m)



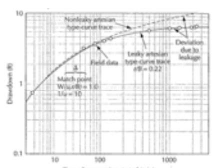
▲ FIGURE 5.11
Type curves of leaky artesian aquifer in which no water is released from storage in the confining layer. Source: W. C. Walton, Illinois State Water Survey Bulletin 49, 1962.

Leaky Confined Aquifer: Example

Table 5.2

Time (min)	Drawdown (ft)
5	0.76
28	3.30
41	3.59
60	4.08
75	4.39
244	5.47
493	5.96
669	6.11
958	6.27
1129	6.40
1185	6.42

► FIGURE 5.12
Field-data plot of drawdown as a function of time for a leaky, confined aquifer. Source: W. C. Walton, Illinois State Water Survey Bulletin 49, 1962.



Leaky Confined Aquifer: Example

Walton (1960) gave time-drawdown data for an aquifer test in a well confined by a stratum of silty fine sand that was 14 ft thick (Table 5.2). Drawdown was measured in an observation well 96 ft from the pumping well, which was pumped at 25 gal/min. Using Walton's graphical method for the Hantush-Jacob formulas, find the values of T, S, and K'.

First a plot of drawdown as a function of time must be made. This is shown in Figure 5.12. The time-drawdown data are matched to an r/B curve. The match-point values are:

$$W(u, r/B) = 1.0$$
$$1/u = 10, \qquad u = 0.10$$
$$h_0 - h = 1.9 \text{ ft}$$
$$t = 33 \text{ min}$$
$$r/B = 0.22$$

Next we must convert gallons per minute to cubic feet per day and time in minutes to time in days.

$$Q = 25 \text{ gal/min} \times 1/7.48 \text{ ft}^3/\text{gal} \times 1440 \text{ min/d} = 4800 \text{ ft}^3/\text{d}$$

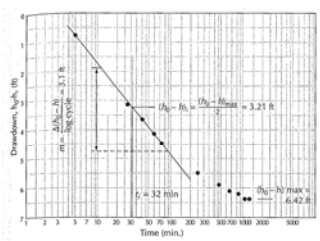
$$t = 33 \text{ min} \times 1/1440 \text{ d/min} = 0.023 \text{ d}$$

Leaky Confined Aquifer: Example

The match-point values, Q and r, are substituted into Equations 5.60, 5.61, and 5.63.

$$T = \frac{Q}{4\pi(h_0 - h)} W(u, r/B)$$
$$= \frac{4800\ \text{ft}^3/\text{d}}{4 \times \pi \times 1.9\ \text{ft}} \times 1$$
$$= 200\ \text{ft}^2/\text{d}$$
$$S = \frac{4Tut}{r^2}$$
$$= \frac{4 \times 200\ \text{ft}^2/\text{d} \times 0.10 \times 0.023\ \text{d}}{96\ \text{ft} \times 96\ \text{ft}}$$
$$= 0.00020$$
$$K' = \frac{Tb'(r/B)^2}{r^2}$$
$$= \frac{200\ \text{ft}^2/\text{d} \times 14\ \text{ft} \times 0.22^2}{96\ \text{ft} \times 96\ \text{ft}}$$
$$= 0.015\ \text{ft/d}$$

Hantush Inflection Point Method



▲ FIGURE 5.13
Plot of drawdown in a confined aquifer as a function of time on semilogarithmic paper for use in the Hantush inflection-point method of analysis.

Table 5.2

Time (min)	Drawdown (ft)
5	0.76
28	3.30
41	3.59
60	4.08
75	4.39
244	5.47
493	5.96
669	6.11
958	6.27
1129	6.40
1185	6.42

Hantush Inflection Point Method

The following relations hold true for the inflection point:

$$u_i = \frac{r^2S}{4t_iT} = \frac{r}{2B} \quad (5.64)$$

$$m_i = \left(\frac{2.3Q}{4\pi T}\right)\exp\left(\frac{-r}{B}\right) \quad (5.65)$$

$$(h_0 - h)_i = 0.5\,(h_0 - h)_{max} = \frac{Q}{4\pi T}K_0\,(r/B) \quad (5.66)$$

$$B = \left(\frac{T}{K'/b'}\right)^{1/2} \quad (5.67)$$

$$f\left(\frac{r}{B}\right) = \frac{2.3(h_0 - h)_i}{m_i} = \exp\left(\frac{r}{B}\right)K_0\left(\frac{r}{B}\right) \quad (5.68)$$

where K_0 is a function with values tabulated in Appendix 5 as $K_0(x)$ and $\exp(x)K_0(x)$.

From the drawdown and slope at the inflection point, the value of $f(r/B)$ may be found:

$$f(r/B) = 2.3(h_0 - h)_i/m_i \quad (5.69)$$

Knowing the value of $f(r/B)$, the function tables may be used to find the value of r/B, since $f(x) = \exp(x)K_0(x)$. Since r is known, the value of B may easily be found.

The transmissivity may be found from the relation

$$T = \frac{QK_0(r/B)}{2\pi(h_0 - h)_{max}} \quad (5.70)$$

Leaky Confined with Storage

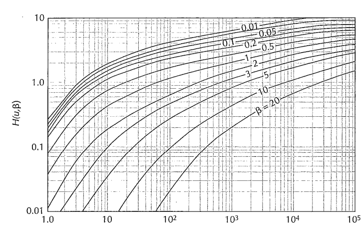
From the match-point value of $H(u, \beta)$ and h_0–h, the value of T is found from:

$$T = \frac{Q}{4\pi(h_0 - h)} H(u, \beta) \qquad \textbf{(5.73)}$$

and S may be found from the match-point values of t, $1/u$, the measured value of r, and the computed value of T:

$$S = \frac{4Tut}{r^2} \qquad \textbf{(5.74)}$$

Leaky Confined with Storage



▲ FIGURE 5.14
Type curves for a well in an aquifer confined by a leaky layer that releases water from storage.
Source: Data from M. S. Hantush, Journal of Geophysical Research *65 (1960): 3713–25; type curves from W. C. Walton,* Groundwater Resource Evaluation *(New York: McGraw-Hill, 1970).*

Leaky Confined with Storage

The value of β can be used to compute the product $K'S'$:

$$\beta^2 = \frac{r^2 S'}{16B^2 S} \qquad \textbf{(5.75)}$$

$$B^2 = T/(K'/b') \qquad \textbf{(5.76)}$$

Combining Equations 5.75 and 5.76,

$$K'S' = \frac{16\beta^2 T b' S}{r^2} \qquad \textbf{(5.77)}$$

If one of the values, either K' or S', is known, the value of the other can be found.

Unconfined Aquifer

The flow equation for unconfined aquifers is

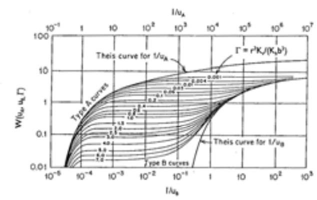
$$T = \frac{Q}{4\pi(h_0 - h)} W(u_A, u_B, \Gamma)] \qquad (5.78)$$

where $W(u_A, u_B, \Gamma)$ is the well function for the water-table aquifer and

$$S = \frac{4Tu_A t}{r^2} \text{ (for early drawdown data)} \qquad (5.79)$$

$$S_y = \frac{4Tu_B t}{r^2} \text{(for later drawdown data)} \qquad (5.80)$$

Unconfined Aquifer



▲ FIGURE 5.15
Type curves for drawdown data from fully penetrating wells in an unconfined aquifer.
Source: S. P. Neuman, Water Resources Research *11 (1975):329–42. Used with permission.*

Unconfined Aquifer

$$\Gamma = \frac{r^2 K_v}{b^2 K_h} \qquad (5.81)$$

where

- h_0–h is the drawdown (L; ft or m)
- Q is the pumping rate (L^3/T; ft^3/d or m^3/d)
- T is the transmissivity (L^2/T; ft^2/d or m^2/d)
- r is the radial distance from the pumping well (L; ft or m)
- S is the storativity (dimensionless)
- S_y is the specific yield (dimensionless)
- t is the time (T; d)
- K_h is the horizontal hydraulic conductivity (L/T; ft/d or m/d)
- K_v is the vertical hydraulic conductivity (L/T; ft/d or m/d)
- b is the initial saturated thickness of the aquifer (L; ft or m)

value of Γ comes from the type curve. The value of T is found using these values and Equation 5.78. The storativity is found from Equation 5.79.

2. The latest drawdown data are then superposed on the Type-B curve for the Γ value of the previously matched Type-A curve. A new set of match points is determined. The value of T calculated from Equation 5.78 should be approximately equal to that computed from the Type-A curve. Equation 5.80 can be used to compute the specific yield.
3. The value of the horizontal hydraulic conductivity can be determined from

$$K_h = T/b \tag{5.82}$$

4. The value of the vertical hydraulic conductivity can also be computed using the Γ value of the matched type curve. Rearrangement of Equation 5.81 yields the following formula:

$$K_v = \frac{\Gamma b^2 K_h}{r^2} \tag{5.83}$$

Unconfined Aquifer Example

A well pumping at 1000 gal/min fully penetrates an unconfined aquifer with an initial saturated thickness of 100 ft. Time-drawdown data for a well located 200 ft away are plotted on log paper (Figure 5.16). Find T, S_y, S, K_h, and K_v.

The value of the pumping rate is

$$Q = 1000 \text{ gal/min} \times 1/7.48 \text{ ft}^3/\text{gal} \times 1440 \text{ min/d} = 1.9 \times 10^5 \text{ ft}^3/\text{d}$$

The early-time drawdown data fit best on the Type-A curve for Γ = 0.1. The selected match point is $W(u_A, \Gamma) = 0.1$, $1/u_A = 1.0$, $h_0 - h = 0.041$ ft, and $t = 0.9$ min. The value of u_A is 1.0 and $t = 6.25 \times 10^{-4}$ d. From Equation 5.78,

$$T = \frac{Q}{4\pi(h_0 - h)} W(u_A, \Gamma)$$

$$= \frac{1.9 \times 10^5 \text{ ft}^3/\text{d} \times 0.1}{4 \times \pi \times 0.041 \text{ ft}}$$

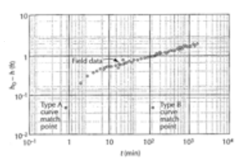
$$= 3.7 \times 10^4 \text{ ft}^2/\text{d}$$

The storativity value is found from Equation 5.79.

$$S = \frac{4Tu_A t}{r^2}$$

$$= \frac{4 \times 3.7 \times 10^4 \text{ ft}^2/\text{d} \times 6.25 \times 10^{-4} \text{ d} \times 1.0}{(200 \text{ ft})^2}$$

$$= 0.0023$$



Unconfined Aquifer Example

The later-time-drawdown data are now matched to the Type-B curve for Γ = 0.1. With the axes of the two sheets of graph paper parallel, the selected match point has values of $W(u_B, \Gamma) = 0.1$, $1/u_B = 10$, $h_0 - h = 0.043$ ft, and $t = 128$ min. The value of u_B is 0.1 and $t = 0.089$ d. From Equation 5.78,

$$T = \frac{Q}{4\pi(h_0 - h)} W(u_B, \Gamma)$$

$$= \frac{1.9 \times 10^5 \text{ ft}^3/\text{d} \times 0.1}{4 \times \pi \times 0.043 \text{ ft}}$$

$$= 3.5 \times 10^4 \text{ ft}^2/\text{d}$$

The value of the specific yield can be determined from Equation 5.80:

$$S_y = \frac{4Tu_B t}{r^2}$$

$$= \frac{4 \times 3.5 \times 10^4 \text{ ft}^2/\text{d} \times 0.089 \text{ d} \times 0.1}{(200 \text{ ft})^2}$$

$$= 0.031$$

The value of the horizontal hydraulic conductivity can be found from Equation 5.82. The average of T is 3.6×10^4 ft^2 per d:

$$K_h = T/b$$

$$= (3.6 \times 10^4 \text{ ft}^2/\text{d})/100 \text{ ft}$$

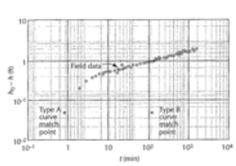
$$= 360 \text{ ft/d}$$

and the value of K_v is determined using Equation 5.83:

$$K_v = \frac{\Gamma b^2 K_h}{r^2}$$

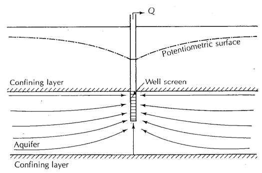
$$= \frac{0.1 \times (100 \text{ ft})^2 \times 360 \text{ ft/d}}{(200 \text{ ft})^2}$$

$$= 9.0 \text{ ft/d}$$



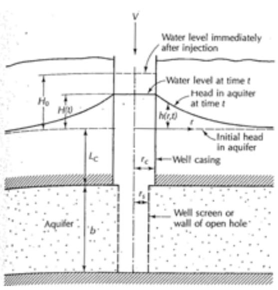
Well screened in part of aquifer

1. If two observation wells equidistant from the pumping well are screened in different parts of the aquifer, the time-drawdown curves may be different.



2. Depending upon the length and relative position of observation-well screens, it is possible for a more distant well to have a greater drawdown than a closer well.
3. The effects of partial penetration produce a time-drawdown curve similar in shape to one produced when there is a downward leakage from storage through a thick, semipervious layer.
4. Partial-penetration effects may produce a time-drawdown curve that resembles the effect of a recharge boundary, a fully penetrating well in either a sloping water-table aquifer or an aquifer of nonuniform thickness.

Slug Tests



◄ FIGURE 5.18
Well into which a volume, *V*, of water is suddenly injected for a slug test of a confined aquifer. *Source: H. H. Cooper, Jr., J. D. Bredehoeft, & I. S. Papadopulos,* Water Resources Research *3 (1967): 263–9. Used with permission.*

Table 5.3

Time (s)	H (m)	H/H_0
2	0.37	0.88
5	0.34	0.81
10	0.27	0.64
21	0.18	0.43
46	0.09	0.21
70	0.05	0.12
100	0.02	0.05

Slug Tests

A plot of the ratio of the measured head to the head after injection (H/H_0) is made as a function of time. The ratio H/H_0 is on the arithmetic scale, and time is on the logarithmic scale of semilogarithmic paper. The ratio H/H_0 is equal to a defined function:

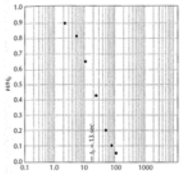
$$H/H_0 = F(\eta, \mu) \quad (5.85)$$

where

$$\eta = Tt/r_c^2 \quad (5.86)$$

and

$$\mu = r_s^2 S/r_c^2 \quad (5.87)$$



◄ FIGURE 5.20
Field-data plot of H/H_0 as a function of time for a slug-test analysis.

Slug Tests

The field-data curve is placed over the type curves with the arithmetic axis coincident. That is, the value of $H/H_0 = 1$ for the field data lies on the horizontal axis of 1.0 on the type curve. The data are matched to the type curve (μ), which has the same curvature. The vertical time-axis, t_1, which overlays the vertical axis for $Tt/r_c^2 = 1.0$, is selected. The transmissivity is found from

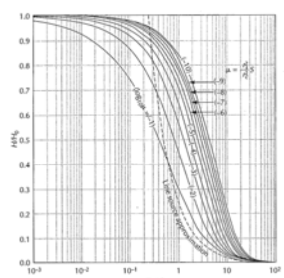
$$T = \frac{1.0 r_c^2}{t_1} \quad (5.88)$$

The value of storativity can be found from the value of the μ-curve for the field data. Since $\mu = (r_s^2/r_c^2)S$,

$$S = (r_c^2 \mu)/r_s^2 \quad (5.89)$$

For small values of μ, however, the curves are often very similar; therefore, in matching the field data, the question of which μ value to use is often encountered. The use of this method to estimate storativity should be approached with caution. Likewise, the value of T that is determined is representative of the formation only in the immediate vicinity of the test hole.

Slug Tests: Type Curves



▲ FIGURE 5.19
Type curves for slug test in a well of finite diameter. *Source: S. S. Papadopulos, J. D. Bredehoeft, and H. H. Cooper, Jr.,* Water Resources Research *9 (1973): 1087–89. Used with permission.*

Slug Tests: Example

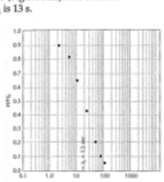
A casing with a radius of 7.6 cm is installed through a confining layer. A screen with a radius of 5.1 cm is installed in a formation with a thickness of 5 m. A slug of water is injected, raising the water level 0.42 m. The decline of the head is given in Table 5.3. Find the values of T, K, and S.

A plot of H/H_0 as a function of t is made on semilogarithmic paper (Figure 5.20). It is overlain on the type curve (Figure 5.19). At the axis for $Tt/r^2 = 1.0$, the value t_1 is 13 s.

$$\begin{aligned} T &= \frac{1.0 r_c^2}{t_1} \\ &= \frac{1.0 \times (7.6\ \text{cm})^2}{13\ \text{s}} \\ &= 4.4\ \text{cm}^2/\text{s} \\ K &= T/b \\ &= 4.4\ \text{cm}^2/\text{s}/500\ \text{cm} \\ &= 8.8 \times 10^{-3}\ \text{cm/s} \end{aligned}$$

The μ-curve is 10^{-3}. With $r_c = 7.6$ cm and $r_s = 5.1$ cm, we find

$$\begin{aligned} S &= (\mu r_c^2)/r_s^2 \\ &= 10^{-3} \times (7.6\ \text{cm})^2/(5.1\ \text{cm})^2 \\ &= 2.2 \times 10^{-3} \end{aligned}$$



◄ FIGURE 5.20
Field-data plot of H/H_0 as a function o
time for a slug-test analysis.

Table 5.3

Time (s)	H (m)	H/H_0
2	0.37	0.88
5	0.34	0.81
10	0.27	0.64
21	0.18	0.43
46	0.09	0.21
70	0.05	0.12
100	0.02	0.05

Specific Capacity Data

Mace (1997) employed a similar approach to the analysis of specific-capacity data with transmissivity data from 71 wells in the karstic Edwards Aquifer of Texas. He found the following relationship with a correlation coefficient of 0.891:

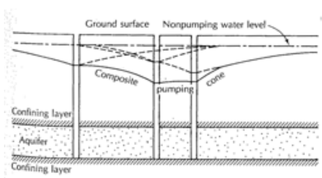
$$T = 0.76\left(\frac{Q}{h_0 - h}\right)^{1.08} \qquad \textbf{(5.106)}$$

where

- T is transmissivity (m^2/d)
- Q is pumping rate (m^3/d)
- h_0–h is drawdown (m)

Empirical Relationship but Specific Capacity Data Routinely Collected

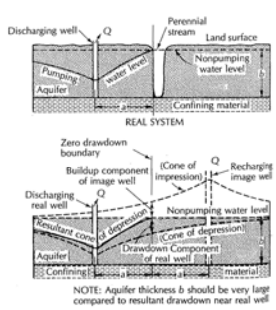
Intersecting Wells: Superposition



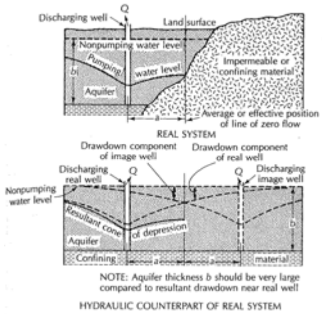
◄ FIGURE 5.29
Composite pumping cone for three wells tapping the same aquifer. Each well is pumping at a different rate; thus the pumping level of each is different.

Sources/Sinks: Superposition

► FIGURE 5.30
Idealized cross section of a well in an aquifer bounded on one side by a stream. *Source: J. G. Ferris et al.*, U.S. Geological Survey Water-Supply Paper 1536-E, *1962.*

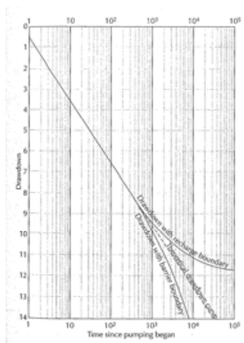


Sources/Sinks: Superposition



◄ FIGURE 5.31
Idealized cross section of a well in an aquifer bounded on one side by an impermeable boundary.
Source: J. G. Ferris et al., U.S. Geological Survey Water-Supply Paper 1536-E, 1962.

Impact of Sources/Sinks



◄ FIGURE 5.32
Impact of recharge and barrier boundaries on semilogarithmic drawdown-time curves.