Unit Conversion

Table 3.1 English and SI Units

Parameter	English Unit	SI Unit	Conversion Factor	Dimensional Formula
Force	pound (lb)	newton (N)	1 lb = 4.448 N	ML/T^2
Mass	slug	kilogram (kg)	1 slug = 14.594 kg	M
Length	foot (ft)	meter (m)	1 ft = 0.3048 m	L
Time	second (s)	second	1 s = 1 s	T
Density	slug/ft^3	kg/m^3	1 slug/ft^3 = 515.4 kg/m^3	M/L^3
Specific weight	lb/ft^3	N/m^3	1 lb/ft^3 = 157.1 N/m^3	M/L^2T^2
Pressure	lb/ft^2	N/m^2	1 lb/ft^2 = 47.88 N/m^2	M/LT^2
Dynamic viscosity	lb-s/ft^2	N·s/m^2	1 lb-s/ft^2 = 47.88 N·s/m^2	M/LT
Bulk modulus	lb/ft^2	N/m^2	1 lb/ft^2 = 47.88 N/m^2	M/LT^2

Porosity

3.2.1 Definition of Porosity

The **porosity** of earth materials is the percentage of the rock or soil that is void of material. It is defined mathematically by the equation

$$n = \frac{100V_v}{V} \quad \textbf{(3.8)}$$

where

n is the porosity (percentage)

V_v is the volume of void space in a unit volume of earth material (L^3; cm^3 or m^3)

V is the unit volume of earth material, including both voids and solids (L^3; cm^3 or m^3)

Porosity Measurement

The total porosity can be computed from the relationship

$$n = 100\,[1 - (\rho_b/\rho_d)] \quad \textbf{(3.9)}$$

where

n is the total porosity as a percentage

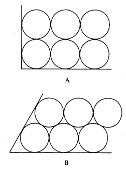
ρ_b is the bulk density of the aquifer material (M/L^3; g/cm^3 or kg/m^3)

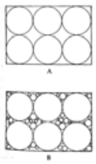
ρ_d is the particle density of the aquifer material (M/L^3; g/cm^3 or kg/m^3)

The bulk density of the aquifer material is the mass of the sample after oven drying divided by the original sample volume (the sample can change volume upon oven drying). The particle density is the oven-dried mass divided by the volume of the mineral matter in the sample as determined by a water displacement test. For most rock and soil the particle density is about 2.65 g/cm^3 (2650 kg/m^3), which is the density of quartz.

Packing and Sorting

► FIGURE 3.1
A. Cubic packing of spheres with a porosity of 47.65%
B. Rhombohedral packing of spheres with a porosity of 25.95%.





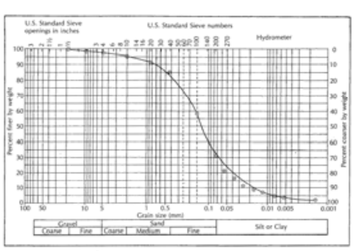
◄ FIGURE 3.2
A. Cubic packing of spheres of equal diameter with a porosity of 47.65 percent. **B.** Cubic packing of spheres with void spaces occupied by grains of smaller diameter, resulting in a much lower overall porosity.

Grain Size Categories

Table 3.2 Engineering Grain-Size Classification

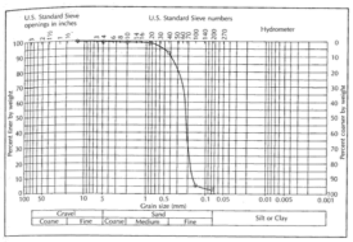
Name	Size range (mm)	Example
Boulder	>305	Basketball
Cobbles	76–305	Grapefruit
Coarse gravel	19–76	Lemon
Fine gravel	4.75–19	Pea
Coarse sand	2–4.75	Water softener salt
Medium sand	0.42–2	Table salt
Fine sand	0.075–0.42	Powdered sugar
Fines	<0.075	Talcum powder

Grain Size Distribution: silty sand



▲ FIGURE 3.4
Grain-size distribution curve of a silty fine to medium sand.

Grain Size Distribution: fine sand



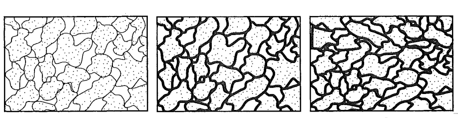
▲ FIGURE 3.5
Grain-size distribution curve of a fine sand.

Table 3.4 Porosity Ranges for Sediments

Well-sorted sand or gravel	25–50%
Sand and gravel, mixed	20–35%
Glacial till	10–20%
Silt	35–50%
Clay	33–60%

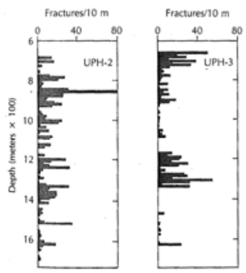
Based on Meinzer (1923a); Davis (1969); Cohen (1965); and MacCary and Lambert (1962).

Reduction in Porosity during Diagenesis



▲ FIGURE 3.6
A. A clastic sediment with intergranular porosity. **B.** Reduction of porosity in the clastic sediment due to deposition of cementing material in the pore spaces. **C.** Further reduction in porosity due to compaction and cementation.

Fracture Porosity



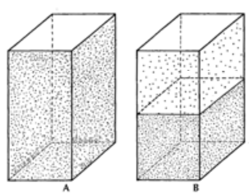
◄ FIGURE 3.7
Histograms showing number of fractures as a function of depth at two deep core holes into Precambrian-age granite in northern Illinois. *Source: B. C. Haimson & T. W. Doe, "State of Stress, Permeability, and Fractures in the Precambrian Granite of Northern Illinois,"* Journal of Geophysical Research *88, B9 (1983): 7355–71.*

3.3 Specific Yield

Specific yield (S_y) is the ratio of the volume of water that drains from a saturated rock owing to the attraction of gravity to the total volume of the rock (Meinzer 1923b) (Figure 3.8).

Water molecules cling to surfaces because of surface tension of the water (Figure 3.9). If gravity exerts a stress on a film of water surrounding a mineral grain, some of the film will pull away and drip downward. The remaining film will be thinner, with a greater surface tension so that, eventually, the stress of gravity will be exactly balanced by the surface tension. **Pendular water** is the moisture clinging to the soil particles because of surface tension. At the moisture content of the specific yield, gravity drainage will cease.

► FIGURE 3.8
A. A volume of rock saturated with water.
B. After gravity drainage, 1 unit volume of the rock has been dewatered with a corresponding lowering of the level of saturation. Specific yield is the ratio of the volume of water that drained from the rock, owing to gravity, to the total rock volume.



Surface Tension

◄ FIGURE 3.9
Pendular water clinging to spheres owing to surface tension. Gravity attraction is pulling the water downward.

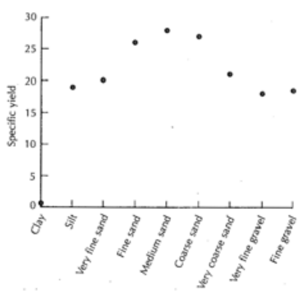
Table 3.5 Specific Yields in Percent

Material	Specific Yield Maximum	Minimum	Average
Clay	5	0	2
Sandy clay	12	3	7
Silt	19	3	18
Fine sand	28	10	21
Medium sand	32	15	26
Coarse sand	35	20	27
Gravelly sand	35	20	25
Fine gravel	35	21	25
Medium gravel	26	13	23
Coarse gravel	26	12	22

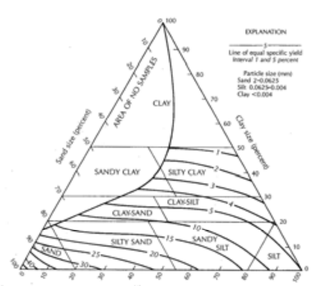
Source: Johnson (1967).

Specific Yield versus Grain Size

▶ FIGURE 3.10
Specific yield of sediments from the Humboldt River Valley of Nevada as a function of the median grain size. *Source: Data from P. Cohen,* U.S. Geological Survey Water-Supply Paper *1975, 1965.*



Specific Yield versus Grain Size



▲ FIGURE 3.11
Textural classification triangle for unconsolidated materials showing the relation between particle size and specific yield. *Source: A. I. Johnson,* U.S. Geological Survey Water-Supply Paper *1662-D, 1967.*

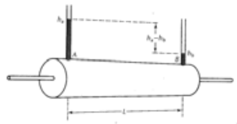
Darcy's Law

Darcy found experimentally that the discharge, Q, is proportional to the difference in the height of the water, h (hydraulic head), between the ends and inversely proportional to the flow length, L:

$$Q \propto h_A - h_B \text{ and } Q \propto -1/L$$

The flow is also obviously proportional to the cross-sectional area of the pipe, A. When combined with the proportionality constant, K, the result is the expression known as **Darcy's law:**

$$Q = -KA\left(\frac{h_A - h_B}{L}\right) \quad \textbf{(3.12)}$$



▲ FIGURE 3.12
Horizontal pipe filled with sand to demonstrate Darcy's experiment. (Darcy's original equipment was actually vertically oriented.)

This may be expressed in more general terms as

$$Q = -KA\left(\frac{dh}{dl}\right) \quad \textbf{(3.13)}$$

where dh/dl is known as the **hydraulic gradient.** The quantity dh represents the change in

3.4.2 Hydraulic Conductivity

We can rewrite Equation 3.13 as

$$q = -K\frac{dh}{dl} \quad \textbf{(3.14)}$$

where $q = Q/A$. The factor q is called the **specific discharge** and has the dimensions of length/time (L/T). It is also sometimes called the **Darcian velocity.** It is not a true velocity as the cross-sectional area, A, is partially blocked with soil material.

Figure 3.13 shows some of Darcy's original data with specific discharge plotted against hydraulic gradient for two different sands. Note that the data fall on straight lines, indicating proportionality; K is the slope of the straight line with dimensions of L/T.

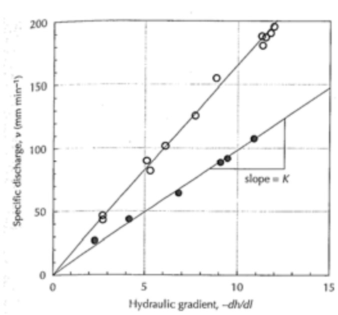
We can also rearrange Equation 3.13 to demonstrate that the coefficient K has the dimensions of L/T. This coefficient has been termed the **hydraulic conductivity.** In other works, it may be referred to as the coefficient of permeability:

$$K = \frac{-Q}{A(dh/dL)} \quad \textbf{(3.15)}$$

Discharge has the dimensions volume/time (L^3/T), area (L^2), and gradient (L/L). Substituting these dimensions into Equation 3.15, the dimensions of K are determined:

$$K = \frac{-(L^3/T)}{(L^2)(L/L)} = (L/T)$$

Darcy's Data



◄ FIGURE 3.13
Original data from Darcy's 1856 experiments that show a linear relationship between specific discharge and hydraulic gradient for two different sands. *Source: Hornberger, Raffensperger, Wiberg and Eshleman,* Elements of Physical Hydrology. *© 1998. The Johns Hopkins University Press. Used with Permission.*

Hydraulic Conductivity (K) – depends on medium and fluid

Hubbert (1956) pointed out that Darcy's proportionality constant, *K*, is a function of properties of both the porous medium and the fluid passing through it. It is intuitively obvious that a viscous fluid (one that is thick), such as crude oil, will move at a slower rate than water, which is thinner and has a lower viscosity. The discharge is directly proportional to the **specific weight,** γ, of the fluid. The specific weight is the force exerted by gravity on a unit volume of the fluid. This represents the driving force of the fluid. Discharge is also inversely proportional to the *dynamic viscosity* of the fluid, μ, which is a measure of the resistance of the fluid to the shearing that is necessary for fluid flow.

If experiments are performed with glass spheres of uniform diameter, the discharge is also proportional to the square of the diameter of the glass beads, *d*.

These proportionality relationships can be expressed as

$$Q \propto d^2$$

$$Q \propto \gamma$$

$$Q \propto \frac{1}{\mu}$$

Intrinsic Permeability (K_i or k) – medium only

Darcy's law can also be expressed as

$$Q = -\frac{Cd^2\gamma A}{\mu}\frac{dh}{dl} \quad \textbf{(3.16)}$$

The new proportionality constant, *C*, is called the *shape factor*. Both *C* and d^2 are properties of the porous media, whereas γ and μ are properties of the fluid. We can introduce a new constant, K_i, which is representative of the properties of the porous medium alone. It is termed the **intrinsic permeability.** This is basically a function of the size of the openings through which the fluid moves. The larger the square of the mean pore diameter, *d*, the lower the flow resistance. The cross-sectional area of a pore is also a function of the shape of the opening. A constant can be used to describe the overall effect of the shape of the pore spaces. Using this dimensionless constant, *C*, the intrinsic permeability is given by the expression

$$K_i = Cd^2 \quad \textbf{(3.17)}$$

Relationship between K and K_i

The dimensions of K_i are (L^2), or area. The relationship between hydraulic conductivity and intrinsic permeability is

$$K = K_i\left(\frac{\gamma}{\mu}\right) \quad \textbf{(3.18)}$$

or

$$K = K_i\left(\frac{\rho g}{\mu}\right) \quad \textbf{(3.19)}$$

where *g* is the acceleration of gravity and ρ is the density.

Units for K_i

Units for K_i can be in square feet, square meters, or square centimeters. In the petroleum industry, the *darcy* is used as a unit of intrinsic permeability. (The petroleum engineer is similarly concerned with the occurrence and movement of fluids through porous media.) The darcy is defined as

$$1 \text{ darcy} = \frac{\frac{1 \text{ cP} \times 1 \text{ cm}^3/\text{s}}{1 \text{ cm}^2}}{1 \text{ atm}/1 \text{ cm}}$$

where

cP is centipoise (a unit of viscosity)

atm is atmosphere (a unit of pressure)

This expression can be converted to square centimeters, since

$$1 \text{ cP} = 0.01 \text{ dyn·s/cm}^2$$

and

$$1 \text{ atm} = 1.0132 \times 10^6 \text{ dyn/cm}^2$$

Substituting into the definition of the darcy, it may be seen that

$$1 \text{ darcy} = 9.87 \times 10^{-9} \text{ cm}^2 \approx 10^{-8} \text{ cm}^2$$

Variation in K_i is 9 or more orders of magnitude

Table 3.7 Ranges of Intrinsic Permeabilities and Hydraulic Conductivities for Unconsolidated Sediments

Material	Intrinsic Permeability (darcys)	Hydraulic Conductivity (cm/s)
Clay	$10^{-6}-10^{-3}$	$10^{-9}-10^{-6}$
Silt, sandy silts, clayey sands, till	$10^{-3}-10^{-1}$	$10^{-6}-10^{-4}$
Silty sands, fine sands	$10^{-2}-1$	$10^{-5}-10^{-3}$
Well-sorted sands, glacial outwash	$1-10^{2}$	$10^{-3}-10^{-1}$
Well-sorted gravel	$10-10^{3}$	$10^{-2}-1$

Be Sure of the Units!

Table 3.6 Conversion Values for Hydraulic Conductivity

1 gal/day/ft²	=	0.0408 m/day
1 gal/day/ft²	=	0.134 ft/day
1 gal/day/ft²	=	4.72×10^{-5} cm/s
1 ft/day	=	0.305 m/day
1 ft/day	=	7.48 gal/day/ft²
1 ft/day	=	3.53×10^{-4} cm/s
1 cm/s	=	864 m/day
1 cm/s	=	2835 ft/day
1 cm/s	=	21,200 gal/day/ft²
1 m/day	=	24.5 gal/day/ft²
1 m/day	=	3.28 ft/day
1 m/day	=	0.00116 cm/s

Size, Sorting and K

1. As the median grain size increases, so does permeability, due to larger pore openings.
2. Permeability will decrease for a given median diameter as the standard deviation of particle size increases. The increase in standard deviation indicates a more poorly sorted sample, so that the finer material can fill the voids between larger fragments. (Figure 3.2B)
3. Coarser samples show a greater decrease in permeability with an increase in standard deviation than do fine samples.
4. Unimodal (one dominant size) samples have a greater permeability than bimodal (two dominant sizes) samples. This is again a result of poorer sorting of the sediment sizes, as the bimodal distribution indicates.

Estimating K from Grain Size

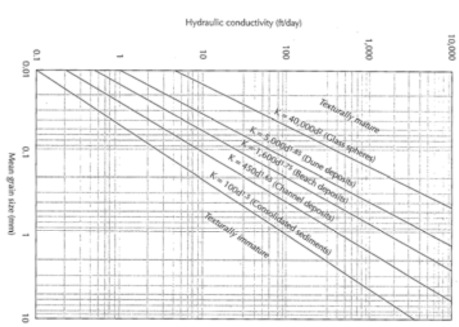
The hydraulic conductivity of sandy sediments can be estimated from the grain-size distribution curve by the **Hazen method** (Hazen 1911). The method is applicable to sands where the effective grain size (d_{10}) is between approximately 0.1 and 3.0 mm. The Hazen approximation is

$$K = C(d_{10})^2 \qquad \textbf{(3.20)}$$

where

K is hydraulic conductivity (cm/s)
d_{10} is the effective grain size (cm)
C is a coefficient based on the following table:

Very fine sand, poorly sorted	40–80
Fine sand with appreciable fines	40–80
Medium sand, well sorted	80–120
Coarse sand, poorly sorted	80–120
Coarse sand, well sorted, clean	120–150



▲ FIGURE 3.15
Graph showing the relationship of hydraulic conductivity to mean grain diameter for sediments of different textural maturity. *Modified from R. G. Shepherd, Ground Water 27, no. 5 (1989): 633–638. Copyright © 1989 Ground Water Publishing Co.*

Hazen Method vs Observation

	Upper Aquifer		Lower Aquifer	
	Mean	Range	Mean	Range
d_{10}	0.14 mm	0.08–0.20 mm	0.16 mm	0.09–0.26 mm
d_{60}	0.31 mm	0.19–0.45 mm	2.04 mm	0.35–6.70 mm
C_u	2.29	1.50–3.89	11.01	3.89–33.50

The hydraulic conductivities of the sediments at each monitoring well were estimated by the Hazen method, using a coefficient of 100. The hydraulic conductivities of the sediments at each monitoring well were measured by means of a Hvorslev slug test performed on the well (see Section 5.6.2.2). The following table compares the results in centimeters per second.

	Geometric Mean (cm/s)	Range (cm/s)
	Upper Aquifer	
Hazen method	1.9×10^{-2}	$4.0 \times 10^{-2} - 6.4 \times 10^{-3}$
Hvorslev test	1.9×10^{-2}	$8.9 \times 10^{-2} - 4.2 \times 10^{-3}$
	Lower Aquifer	
Hazen method	1.2×10^{-2}	$2.6 \times 10^{-2} - 8.1 \times 10^{-3}$
Hvorslev test	1.4×10^{-2}	$1.7 \times 10^{-1} - 2.6 \times 10^{-3}$

Geometric Mean Example

PROBLEM

Find the geometric mean of the following set of hydraulic conductivity values and compare it with the arithmetic mean:

Hydraulic conductivity (K)	ln (K)
2.17×10^{-2} cm/s	−3.83
2.58×10^{-2} cm/s	−3.66
2.55×10^{-3} cm/s	−5.97
1.67×10^{-1} cm/s	−1.79
9.50×10^{-4} cm/s	−6.96
Sum: 2.18×10^{-1} cm/s	−22.21

Geometric mean: mean ln(K): $-22.21/5 = -4.44$

exp [mean ln(K)]: $e^{-4.44} = 1.18 \times 10^{-2}$ cm/s

Arithmetic mean: $(2.18 \times 10^{-1})/5 = 4.36 \times 10^{-2}$ cm/s

Computing K from K_i

PROBLEM

The intrinsic permeability of a consolidated rock is 2.7×10^{-3} darcy. What is the hydraulic conductivity for water at 15°C?

At 15° C for water, from Appendix 14:

$$\rho = 0.999099 \text{ g/cm}^3$$

$$\mu = 0.011404 \ \frac{\text{g}}{\text{s·cm}}$$

The acceleration of gravity is given as

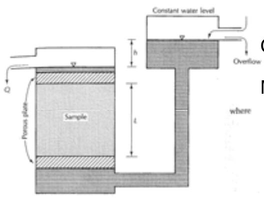
$$g = 980 \text{ cm/s}^2$$

As 1 darcy = 9.87×10^{-9} cm², the intrinsic permeability is 2.66×10^{-11} cm²:

$$K = K_i\left(\frac{\rho g}{\mu}\right) = 2.66 \times 10^{-11} \text{ cm}^2 \times \frac{0.999099 \text{ g/cm}^3 \times 980 \text{ cm/s}^2}{0.011404 \text{ g/s·cm}}$$

$$K = 2.28 \times 10^{-6} \frac{\text{g/cm}^3 \times \text{cm/s}^2 \times \text{cm}^2}{\text{g/s} \cdot \text{cm}}$$

$$= 2.3 \times 10^{-6} \text{ cm/s}$$



Constant Head Permeameter

Measuring K (high values)

where

V is the volume of water discharging in time t (L^3; cm^3, and T; s)
L is the length of the sample (L; cm)
A is the cross-sectional area of the sample (L^2; cm^2)
h is the hydraulic head (L; cm)
K is the hydraulic conductivity (L/T; cm/s)

▲ FIGURE 3.16
Constant-head permeameter apparatus. This is similar to Darcy's original test apparatus.

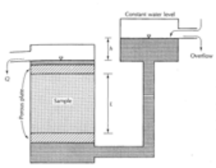
The *constant-head permeameter* is used for noncohesive sediments, such as sand. A chamber with an overflow provides a supply of water at a constant head. Water moves through the sample at a steady rate. The hydraulic conductivity is determined from a variation of Darcy's law, which gives the flux of fluid per unit time, called the discharge, Q. If we collect the fluid draining from a permeameter over some time, t, the total volume, V, is the product of the discharge and time. If we multiply both sides of Equation 3.12 by time, t, and rearrange, we obtain

$$Qt = -\frac{KAt(h_A - h_B)}{L} \quad (3.22)$$

If we substitute V for Qt and use h for $-(h_A - h_B)$, Equation 3.22 can be rearranged to form

$$\boxed{K = \frac{VL}{Ath}} \quad (3.23)$$

Constant Head Example



where

V is the volume of water discharging in time t (L^3; cm^3, and T; s)
L is the length of the sample (L; cm)
A is the cross-sectional area of the sample (L^2; cm^2)
h is the hydraulic head (L; cm)
K is the hydraulic conductivity (L/T; cm/s)

▲ FIGURE 3.16
Constant-head permeameter apparatus. This is similar to Darcy's original test apparatus.

A constant-head permeameter has a sample of medium-grained sand 15 cm in length and 25 cm^2 in cross-sectional area. With a head of 5.0 cm, a total of 100 mL of water is collected in 12 min. Find the hydraulic conductivity.

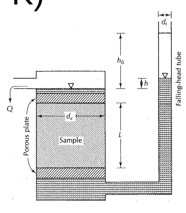
$$K = \frac{VL}{Ath}$$

$$= \frac{100\ cm^3 \times 15\ cm}{25\ cm^2 \times 12\ min \times 60\ s/min \times 5\ cm}$$

$$= 1.7 \times 10^{-2}\ cm/s \text{ or } 14\ m/d$$

Falling Head Permeameter (lower values of K)

▶ FIGURE 3.17
Falling-head permeameter apparatus.



where

K is hydraulic conductivity (L/T; cm/s)
L is sample length (L; cm)
h_0 is initial head in the falling tube (L; cm)
h is final head in the falling tube (L; cm)
t is the time that it takes for the head to go from h_0 to h (T; s)
d_t is the inside diameter of the falling-head tube (L; cm)
d_c is the inside diameter of the sample chamber (L; cm)

The rate at which water will drain from the falling-head tube into the sample chamber is the change in head with time multiplied by the cross-sectional area, A_t, of the falling-head tube.

$$q_{in} = -A_t\frac{dh}{dt} \quad (3.24)$$

If A_c is the cross-sectional area of the sample chamber, we can determine the volume of water draining from the sample chamber from Equation 3.12:

$$q_{out} = \frac{KA_ch}{L} \quad (3.25)$$

Falling Head Permeameter

Under the principle of continuity, the volume of water entering the sample chamber must equal the volume draining from it (i.e., $q_{in} = q_{out}$).

$$-A_t \frac{dh}{dt} = \frac{KA_c h}{L} \quad \textbf{(3.26)}$$

Equation 3.26 can be rearranged to yield:

$$\frac{dh}{h} = -K\frac{A_c}{A_t}\frac{l}{L}dt \quad \textbf{(3.27)}$$

The boundry conditions on this problem are that $h = h_0$ at $t = 0$. If we integrate dh/h on the left side of Equation 3.27 from h_o to h and dt the right side from 0 to t, we can obtain:

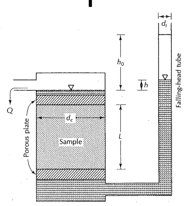
$$1\text{n}\, h - 1\text{n}\, h_o = -K\frac{A_c}{A_t}\frac{1}{L}t \quad \textbf{(3.28)}$$

Equation 3.28 can be rearranged to isolate the hydraulic conductivity, K, on the left side and to eleminate the minus signs. In addition the cross-sectional areas are proportional to the square of the diameters of the falling head tube, d_t, and the samble chamber, d_c. The resulting simplified equation is:

$$\boxed{K = \frac{d_t^2 L}{d_c^2 t}\ln\left(\frac{h_0}{h}\right)} \quad \textbf{(3.29)}$$

Falling Head Example

► FIGURE 3.17
Falling-head permeameter apparatus.



where

K is hydraulic conductivity (L/T; cm/s)
L is sample length (L; cm)
h_0 is initial head in the falling tube (L; cm)
h is final head in the falling tube (L; cm)
t is the time that it takes for the head to go from h_0 to h (T; s)
d_t is the inside diameter of the falling-head tube (L; cm)
d_c is the inside diameter of the sample chamber (L; cm)

A falling-head permeameter containing a silty, fine sand has a falling-head tube diameter of 2.0 cm, a sample diameter of 10.0 cm, and a flow length of 15 cm. The initial head is 5.0 cm. It falls to 0.50 cm over a period of 528 min. Find the hydraulic conductivity.

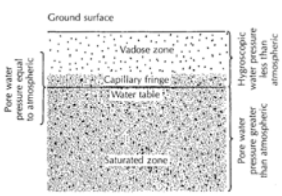
$$K = \frac{d_t^2 L}{d_c^2 t}\ln\left(\frac{h_0}{h}\right)$$

$$= \frac{2.0^2\ \text{cm}^2}{10^2\ \text{cm}^2} \times \frac{15\ \text{cm}}{528\ \text{min} \times 60\ \text{s/min}} \times \ln\frac{5.0\ \text{cm}}{0.50\ \text{cm}}$$

$$= 4.4 \times 10^{-5}\ \text{cm/s or } 3.8 \times 10^{-2}\ \text{m/d}$$

Fluid Pressure at the Water Table

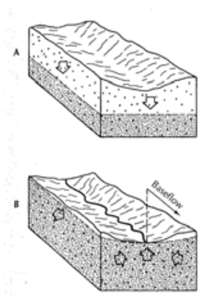
► FIGURE 3.18
Distribution of fluid pressures in the ground with respect to the water table.



Groundwater Flow and the Water Table: Observations

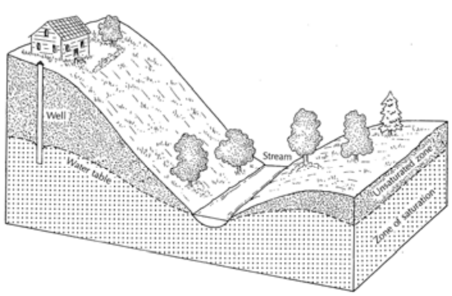
1. In the absence of ground-water flow, the water table will be flat.
2. A sloping water table indicates the ground water is flowing.
3. Ground-water discharge zones are in topographical low spots.
4. The water table has the same general shape as the surface topography.
5. Ground water generally flows away from topographical high spots and toward topographic lows.

Recharge and Discharge



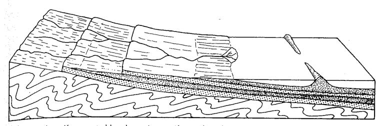
◄ FIGURE 3.19
A. Diagram of a flat-lying water table in an aquifer where there is downward movement of water through the unsaturated zone but no lateral ground-water movement. **B.** Diagram of the water table in a region where water is moving downward through the unsaturated zone to the water table and moving as ground-water flow through the zone of saturation toward a discharge zone along the stream. Net discharge from the aquifer is occurring as baseflow from the stream.

Unconfined Aquifer

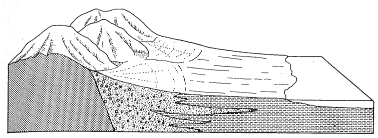


▲ FIGURE 3.20
Unconfined, or water-table, aquifer.

Confined Aquifer

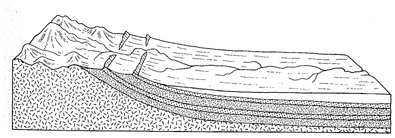


Confined aquifers created by alternating aquifers and confining units deposited on a regional dip.



Confined aquifers created by deposition of alternating layers of permeable sand and gravel and impermeable silts and clays deposited in intermontane basins.

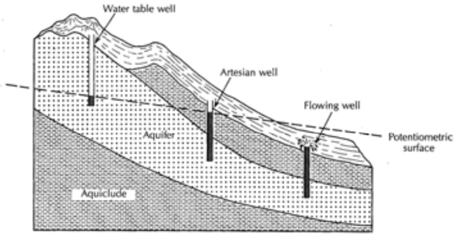
Confined Aquifer: Limited Recharge, capable of abnormal pressure



Confined aquifer created by upwarping of beds by intrusions.

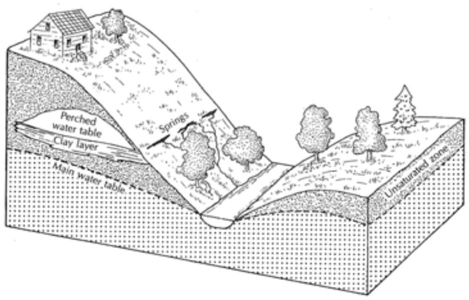
▲ FIGURE 3.21
Confined aquifers created when aquifers are overlain by confining beds.

Potentiometric Surface



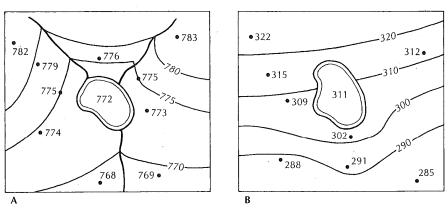
▲ FIGURE 3.22
Artesian and flowing well in confined aquifer.

Perched Aquifer



▲ FIGURE 3.23
Perched aquifer formed above the main water table on a low-permeability layer in the unsaturated zone.

Potentiometric Surface and Surface Water Bodies



▲ FIGURE 3.24
Maps showing construction of water-table maps in areas with surface-water bodies. **A.** A water-table lake with two gaining streams draining into it and one gaining stream draining from it. **B.** A perched lake that, through outseepage, is recharging the water table.

Transmissivity

3.9 Aquifer Characteristics

We have thus far considered the intrinsic permeability of earth materials and their hydraulic conductivity when transmitting water. A useful concept in many studies is aquifer **transmissivity,** which is a measure of the amount of water that can be transmitted horizontally through a unit width by the full saturated thickness of the aquifer under a hydraulic gradient of 1.

The transmissivity is the product of the hydraulic conductivity and the saturated thickness of the aquifer:

$$T = bK \quad \textbf{(3.30)}$$

where

T is transmissivity (L^2/T; ft^2/d or m^2/d)

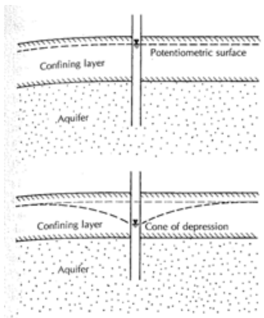
b is saturated thickness of the aquifer (L; ft or m)

K is hydraulic conductivity (L/T; ft/d or m/d)

For a multilayer aquifer, the total transmissivity is the sum of the transmissivity of each of the layers:

$$T = \sum_{i=1}^{n} T_i \quad \textbf{(3.31)}$$

Decline in Head (Pressure) without draining the pores



◄ FIGURE 3.25
Diagram showing lowering of the potentiometric surface in a confined aquifer with the resultant water level still above the aquifer materials. In this circumstance, the aquifer remains saturated.

Specific Storage

The **specific storage** (S_s) is the amount of water per unit volume of a saturated formation that is stored or expelled from storage owing to compressibility of the mineral skeleton and the pore water per unit change in head. This is also called the *elastic storage coefficient*. The concept can be applied to both aquifers and confining units.

The specific storage is given by the following expression (Jacob 1940, 1950; Cooper 1966).

$$S_s = \rho_w g(\alpha + n\beta) \quad \textbf{(3.32)}$$

where

- ρ_w is the density of the water (M/L^3; slug/ft^3 or kg/m^3)
- g is the acceleration of gravity (L/T^2; ft/s^2 or m/s^2)
- α is the compressibility of the aquifer skeleton [$1/(M/LT^2)$; $1/(\text{lb/ft}^2)$ or $1/(\text{N/m}^2)$]
- n is the porosity (L^3/L^3)
- β is the compressibility of the water* ($1/(M/LT^2)$; $1/(\text{lb/ft}^2)$ or $1/(\text{N/m}^2)$)

Specific storage has dimensions of $1/L$. The value of specific storage is very small, generally 0.0001 ft^{-1} or less.

In a confined aquifer, the head may decline—yet the potentiometric surface remains above the unit (Figure 3.25). Although water is released from storage, the aquifer remains saturated. The storativity (S) of a confined aquifer is the product of the specific storage (S_s) and the aquifer thickness (b):

$$S = bS_s \quad \textbf{(3.33)}$$

Storativity

All the water released is accounted for by the compressibility of the mineral skeleton and the pore water. The water comes from the entire thickness of the aquifer. The value of the storativity of confined aquifers is on the order of 0.005 or less.

In an unconfined unit, the level of saturation rises or falls with changes in the amount of water in storage. As the water level falls, water drains from the pore spaces. This storage or release is due to the *specific yield* (S_y) of the unit. Water is also stored or expelled depending on the specific storage of the unit. For an unconfined unit, the storativity is found by the formula

$$S = S_y + bS_s \quad \textbf{(3.34)}$$

where b is the saturated thickness of the aquifer.

The value of S_y is several orders of magnitude greater than bS_s for an unconfined aquifer, and the storativity is usually taken to be equal to the specific yield. For a fine-grained unit, the specific yield may be very small, approaching the same order of magnitude as bS_s. Storativity of unconfined aquifers ranges from 0.02 to 0.30.

Water released or added to storage

The volume of water drained from an aquifer as the head is lowered may be found from the formula

$$V_w = SA\,\Delta h \qquad (3.35)$$

where

V_w is the volume of water drained (L^3; ft^3 or m^3)
S is the storativity (dimensionless)
A is the surface area overlying the drained aquifer (L^2; ft^2 or m^2)
Δh is the average decline in head (L; ft or m)

An unconfined aquifer with a storativity of 0.13 has an area of 123 mi^2. The water table drops 5.23 ft during a drought. How much water was lost from storage?

$$\begin{aligned} V_w &= SA\,\Delta h \\ &= 0.13 \times 123\ mi^2 \times 2.7878 \times 10^7\ ft^2/mi^2 \times 5.23\ ft \\ &= 2.3 \times 10^9\ ft^3 \end{aligned}$$

If the same aquifer had been confined with a storativity of 0.0005, what change in the amount of water in storage would have resulted?

$$\begin{aligned} V_w &= 0.0005 \times 123\ mi^2 \times 2.7878 \times 10^7\ ft^2/mi^2 \times 5.23\ ft \\ &= 9.0 \times 10^6\ ft^3 \end{aligned}$$

Effective Stress

$$\sigma_T = \sigma_e + P$$

where

σ_T is total stress
P is pressure
σ_e is effective stress

If there is a change in total stress, the pressure and effective stress will also change.

$$d\sigma_T = d\sigma_e + dP \qquad (3.37)$$

In confined aquifers, there can be significant changes in pressure with very little change in the actual thickness of the saturated water column. Under these conditions, the total stress remains essentially constant, and any change in pressure will result in a change in effective stress that is of equal magnitude but opposite in sign.

$$dP = -d\sigma_e \qquad (3.38)$$

If pumping reduces the pressure head in a confined aquifer, the effective stress that acts on the aquifer skeleton will increase. The aquifer skeleton may consolidate or compact due to this increased stress. The consolidation occurs due to shifting of the mineral grains, which reduces the porosity.

Aquifer Compressibility

Aquifer compressibility is defined as

$$\alpha = \frac{-\,db/b}{d\sigma_e} \qquad (3.39)$$

where

α is aquifer compressibility [$1/(M/L^2)$; ft^2/lb or m^2/N]
db is change in aquifer thickness (L; ft or m)
b is original aquifer thickness (L; ft or m)
$d\sigma_e$ is change in effective stress (M/LT^2; lb/ft^2 or N/m^2)

The negative sign indicates that the aquifer gets smaller with an increase in effective stress.

Since $dP = -d\sigma_e$, Equation 3.39 can also be written as

$$+\alpha = \frac{db/b}{dP} \qquad (3.40)$$

Aquifer Compressibility - Example

A confined aquifer with an initial thickness of 45 m consolidates (compacts) 0.20 m when the head is lowered by 25 m.

Part A: What is the vertical compressibility of the aquifer?

The given parameter values are $dP = 25$ m, $b = 45$ m, and $db = 0.20$ m. A pressure head of 25 m of water can be converted to a fluid pressure by multiplying the pressure head by the density of water times the gravitational constant.

$$dP = 25 \text{ m} \times 1000 \text{ kg/m}^3 \times 9.8 \text{ m/s}^2 = 245{,}000 \text{ N/m}^2$$

From Equation 3.40,

$$\alpha = \frac{(0.20 \text{ m})/(45 \text{ m})}{245{,}000 \text{ N/m}^2} = 1.8 \times 10^{-8} \text{ m}^2/\text{N}$$

Part B: If the porosity of the aquifer is 12% after compaction, calculate the storativity of the aquifer.

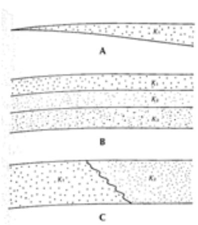
Aquifer storativity is found from Equations 3.32 and 3.33:

$$S = b[\rho_w g(\alpha + n\beta)]$$

The given parameter values are $b = 44.8$ m, $n = 0.12$, $\rho_w = 1000$ kg/m^3, $g = 9.8$ m/s^2, $\alpha = 1.8 \times 10^{-8}$ m^2/N, and $\beta = 4.6 \times 10^{-10}$ m^2/N.

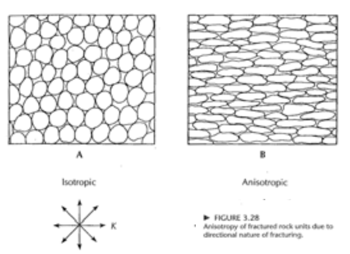
$$S = (44.8 \text{ m})[1000 \text{ kg/m}^3 \times 9.8 \text{ m/s}^2(1.8 \times 10^{-8} \text{ m}^2/\text{N} + 0.12 \times 4.6 \times 10^{-10} \text{ m}^2/\text{N})]$$
$$= (44.8 \text{ m})(9800 \text{ N/m}^3)(1.806 \times 10^{-8} \text{ m}^2/\text{N})$$
$$= 7.9 \times 10^{-3}$$

Heterogeneous Sediments



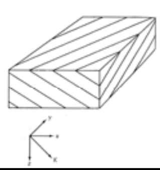
◀ FIGURE 3.26
A. Heterogeneous formation consisting of a sediment that thickens in a wedge. **B.** Heterogeneous formation consisting of three layers of sediments of differing hydraulic conductivity. **C.** Heterogeneous formation consisting of sediments with different hydraulic conductivities lying next to each other.

Anisotropic Sediments

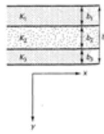


▲ FIGURE 3.27
Grain shape and orientation can affect the isotropy

► FIGURE 3.28
Anisotropy of fractured rock units due to directional nature of fracturing.



► FIGURE 3.29
Heterogeneous formation consisting of three layers of differing hydraulic conductivity.



The average horizontal conductivity (parallel to layering) is found from the summation

$$K_h \text{ avg} = \sum_{m=1}^{n} \frac{K_{hm} b_m}{b} \quad (3.41)$$

where

- K_h avg is the average horizontal hydraulic conductivity (L/T; ft/d or m/d)
- K_{hm} is the horizontal hydraulic conductivity of the mth layer (L/T; ft/d or m/d)
- b_m is the thickness of the mth layer (L; ft or m)
- b is the total aquifer thickness (L; ft or m)

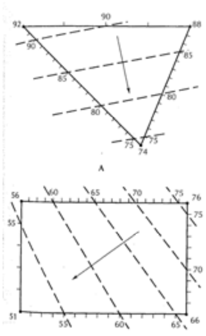
The overall vertical hydraulic conductivity (perpendicular to layering) is given by

$$K_v \text{ avg} = \frac{b}{\sum_{m=1}^{n} \frac{b_m}{K_{vm}}} \quad (3.42)$$

where

- K_v avg is the average vertical hydraulic conductivity (L/T; ft/d or m/d)
- K_{vm} is the vertical hydraulic conductivity of the mth layer (L/T; ft/d or m/d)
- b_m is the thickness of the mth layer (L; ft or m)
- b is the total aquifer thickness (L; ft or m)

Flow Direction from Head Measurements



◄ FIGURE 3.30
Graphical method for determining the slope of a potentiometric surface from **A.** three wells and **B.** four wells.