# Regional Flow Driven by Topographic Recharge

# Topographically Driven Flow

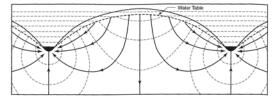
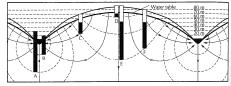


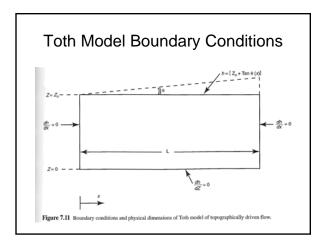
Figure 7.9 Topographically driven flow in a setting with alternating valleys and hills. (From Toth, 1962; after Hubbert, 1940.)

# Topographically Driven Flow



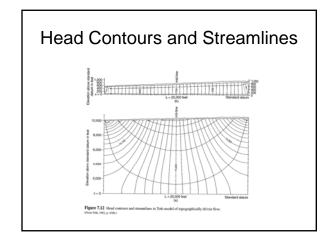
▲ FIGURE 7.2
Plezometers superimposed on Figure 7.1. The water level in the plezometer will rise to the elevation of the hydraulic head, which is represented by the equipotential line at the open end of the piezometer. Source: Modified from M. K. Hubbert, Journal of Geology 48, no. 8 (1940): 793–944. Used with permission of the University of Chicago Processity.

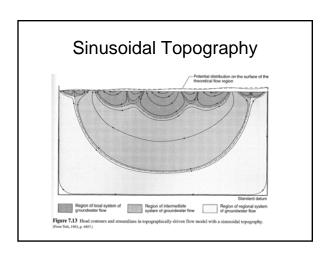
# Toth Model Boundary Conditions Water divide All-water interface, and fluid potential (9) on that interface Theoretical impermeable boundary (Standard datum) Mid-line between bottom and divide Actual region of flow Theoretical and actual region of flow Figure 7.10 Toth model of topographically-driven flow in which alternating hills and valleys are approximated by trapezoids. (9) the 179. 4.179.)

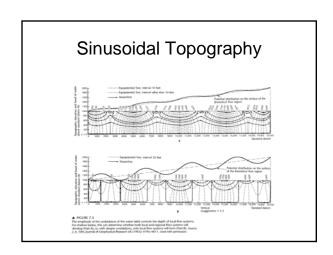


# Toth's Solution to Laplace's Eqn.

 $h=g\left(z_0+\frac{\tan aL}{2}\right)-\frac{4g\tan aL}{\pi^2}\cdot\sum_{n=0}^{\infty}\frac{\cos[(2m+1)\pi x/L]\cosh[(2m+1)\pi x/L)]}{(2m+1)^2\cosh[(2m+1)\pi z_0/L)]} \end{7.1}$  is the head (L)  $g\quad \text{is the gravitational constant } (L/T^2)$   $z_0\quad \text{is the elevation of the water table at its lowest point above the bottom of the aquifer (L) <math display="block">z\quad \text{is the elevation of the water table above the bottom of the aquifer (L)}$   $\tan \alpha \text{ is the slope of the water table}$   $L\quad \text{is the total length of the flow system } (L)$   $z\quad \text{is the horizontal distance from the place where the water table is at its lowest elevation (L)$ 







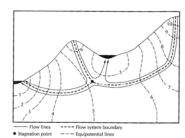
# Effects of Water Table Configuration

### Summary of Water Table Effects

In summary, Freeze and Witherspoon's modeling studies of topographically-driven groundwater flow showed that

- Groundwater discharge tends to be concentrated in major valleys.
   Recharge areas are invariably larger than discharge areas.
   In areas with hummocky terrain, numerous sub-basins are superimposed on the regional flow system.

# Stagnation Point



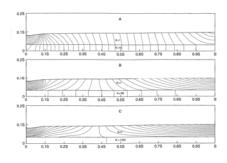
# Laplace's Equation (Variable K)

Laplace's equation in two dimensions,

$$\frac{d\left[-K_x\frac{dh}{dx}\right]}{dx} + \frac{d\left[-K_z\frac{dh}{dz}\right]}{dz} = 0 (7.20)$$

where  $K_{\rm x}$  (m-s<sup>-1</sup>) is the hydraulic conductivity in the horizontal dimension, and  $K_{\rm z}$  (m-s<sup>-1</sup>) is the hydraulic conductivity in the vertical dimension.

# **Aquifer Effects**



# **Aquifer Effects**

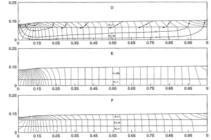
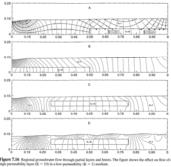


Figure 7.15 Regional groundwater flow through layered media with a simple water-table configuration, but contrasts in hydraulic conductivity between layers. (Pown Pruze and Webreson, 1962, a 627)

# **Aquifer Effects**



# **Summary of Aquifer Effects**

- 4. Buried aquifers tend to concentrate flow toward the principal discharge area, limit the importance of sub-basins in producing small scale flow systems, and need not outcrop to produce artesian flow conditions.
  5. Stratigraphic discontinuities can lead to distributions of recharge and discharge areas that are difficult to anticipate and that are largely independent of the water table configuration.

# **Density Driven Flow**

$$\Delta h = \frac{(\bar{\rho}_1 z_1 - \bar{\rho}_2 z_2)}{\bar{\rho}} \tag{7.22}$$

Differences in fluid density are caused by differences in salinity, temperature, and pressure. All of these factors are variable throughout the Earth's crust. Suppose that the difference between  $\bar{\rho}_1$  and  $\bar{\rho}_2$  is only 1%, such that  $\bar{\rho}_1=1010$  kg·m $^{-3}$ , and  $\bar{\rho}_2=1000$  kg·m $^{-3}$ . Then

$$\Delta h = \frac{10 \text{ (kg-m}^{-3}) \times 5000 \text{ m}}{1005 \text{ (kg-m}^{-3})} = 50 \text{ (m)}$$
 (7.23)

### **Density Driven Flow**

The Darcy Velocity of the horizontal fluid flow between these two points is (from equation 2.55)

$$q = \frac{-k\bar{p}g}{\mu} \nabla h \qquad (7.24)$$

where k (m²) is permeability and μ is fluid viscos-ity (kg-m²-s²). If we assume a typical crustal permeability of 10<sup>-18</sup> m², fluid viscosity of 10<sup>-3</sup> kg-m²-l-s²-l (pure water at 150 °C, equation 4.23), then

$$q = \frac{10^{-15} \text{m}^2 \times 1005 \text{ (kg-m}^{-3}) \times 9.8 \text{ (m-s}^{-2})}{10^{-3} \text{ (kg-m}^{-1} \cdot \text{s}^{-1})} \times \frac{50}{10000} \frac{\text{m}}{\text{m}}$$
(7.25)

$$q = 4.9 \times 10^{-11} \, (\text{m-s}^{-1}) \times 3.156 \times 10^{7} \, (\text{s-yr}^{-1}) = 0.0015 \, (\text{m-yr}^{-1})$$
 (7.26)

### **Density Driven Flow**

Assuming a typical porosity of 10%, the linear velocity is ten times higher than the Darcy velocity (equation 2.19), thus

$$v = 0.015 \text{ (m-yr}^{-1)}$$
 (7.27)

This may not seem like much, but the time it takes to completely exchange the fluid between point 1 and point 2 is now

time = 
$$\frac{\text{distance}}{\text{velocity}} = \frac{10000 \text{ (m)}}{0.015 \text{ (m-yr}^{-1)}} = 6.7 \times 10^5 \text{yr}$$

In other words, there is a complete fluid exchange between points 1 and 2 every 670,000 years.

### Rayleigh Analysis (Free Thermal Convection)

$$R_a = \frac{\alpha_7 g \rho^2 C k y^2 \gamma}{\mu \lambda}$$
 (7.29)

where  $\alpha_T(K^{-1} \text{ or } ^{\circ}C^{-1})$  is the coefficient of thermal expansion for a fluid, g ( $m \cdot s^{-2}$ ) is the acceleration due to gravity,  $\rho$  ( $kg \cdot m^{-3}$ ) is fluid density, C ( $-kg^{-1} \cdot K^{-1}$ ) is fluid specific heat capacity, k ( $m^{\circ}$ ) is permeability, y (m) is height of the porous medium between boundaries, or cell height,  $\gamma$  ( $K \cdot m^{-1}$ ) is the thermal gradient,  $\mu$  ( $kg \cdot m^{-1} \cdot s^{-1}$ ) is the fluid dynamic viscosity, and  $\lambda$  ( $W \cdot m^{-1} K^{-1}$ ) is the thermal conductivity of the saturated porous medium.

# Rayleigh Analysis (Free Thermal Convection)

The critical value of  $R_a$  at which convection begins in the porous medium described above is  $4\Pi^2 \sim 40$  (Turcotte and Schubert, 1982, p. 405). If we apply a Rayleigh analysis to the continuation cust, we may ask: for what value of permeability does convection begin? Rearranging equation 7.29,

$$k = \frac{40\mu\lambda}{\alpha g \rho^2 C y^2 \gamma}$$
(7.1)

Let  $\alpha_T=10^{-3}~{\rm K}^{-1},~g=9.8~{\rm m\cdot s}^{-2},~\rho=1000$  kg·m<sup>-3</sup>,  $C=4200~{\rm J\cdot kg}^{-1}.{\rm K}^{-1},~y=5000~{\rm m},~\gamma=0.025~{\rm K\cdot m}^{-1},~\mu=5\times10^{-4}~{\rm kg\cdot m}^{-1}.{\rm s}^{-1},~{\rm and}~\lambda=2.5~{\rm W\cdot m}^{-1}.{\rm K}^{-1},~{\rm We~then~find~that}$ 

$$k = 1.9 \times 10^{-15} \,\mathrm{m}^2$$
 (7.3)

Compilations of data by Brace (1984) and Classy (1992) show that average permeabilities of crystalline rocks in the continental crust on the kilometer scale are of the order 10<sup>-15</sup> m<sup>2</sup>; thus convection is apparently feasible over a representative range of thicknesses and thermal gradients.

# Rayleigh Analysis (Free Thermal Convection)

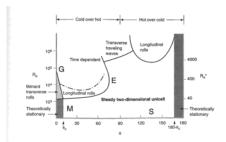


Figure 7.17 Convective flow regimes in a filted porous medium as a function of tilt angle  $(\Theta)$  and Rayleigh number  $(R_i)$ . (Ivon Cins and Hofresiste, 1991, p. 200.)

# Seismic Pumping

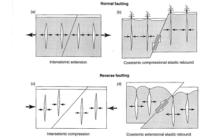


Figure 7.18 (a) In extensional termins, posonily and fluid storage increase as strain builds up between earthquakes, (b) Dring and immediately following an earthquake on a normal fault, strain is released, provily decreases, and fluids are expelled. (c) In compressional termins, peroulty and fluid storage decrease as strain build up between earthquakes (c) During and immediately following an earthquake on a reverse fault, strain is released, promoting increases, and fluids are abourbed.

(Flux Nos. (1994, p. 1804).

# Seismic Pumping The state of t

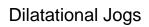




Figure 7.20 Arrows illustrate flow into a dilatational jog immediately following fault movement (After Sibson, 1987, p. 702.)

# Fault Valve Action

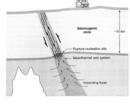
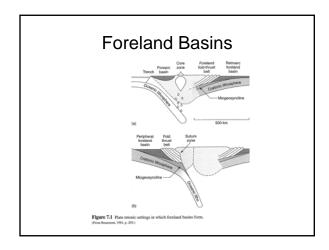
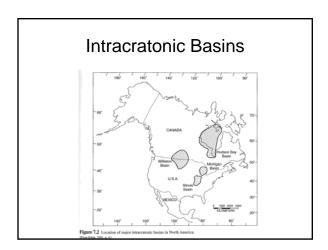
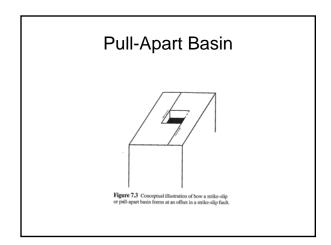




Figure 7.21 Schematic illustration (top) of fault-valve action wherein lithostatically-pressured fluids from the lower continental crust are released into the upper crust by repturing related to a seismic event. Bottom figure sho







### **Sedimentation Rates**

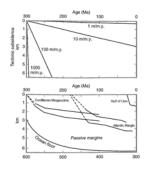
TABLE 7.1	Average Sedimentation Rates.
Geologic Setting	g Sedimentation Rate (m-Ma <sup>-2</sup>
intracratonic basis	ns ~10
rift basins	~50-100
foreland basins	~100
strike-slip/forear	c ~100-1,000
river deltas	~1,000-10,000

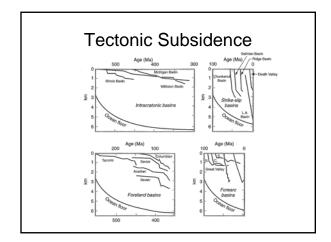
# Compaction Driven Flow

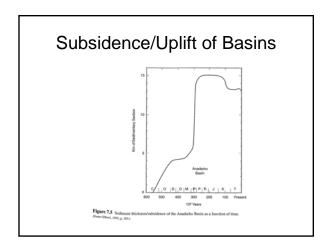
$$q = \frac{0.5RA}{A} = 0.5R \tag{7.3}$$

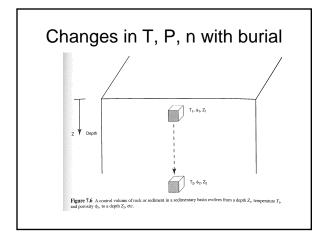
In other words, the average Darcy velocity of fluid flow due to sediment compaction is proportional to the sedimentation rate and is of the same order of magnitude. Sedimentation rate varies with the geologic setting (Table 7.1).

# Tectonic Subsidence









# Compaction vs. Aquathermal Pressure

$$\frac{d\rho}{\rho} = BdP - \alpha_T dT \tag{7.4}$$

where  $\rho$  (kg·m<sup>-2</sup>) is fluid density, B (m·s<sup>2</sup>-kg<sup>-1</sup>) is the isothermal fluid compressibility (as defined in equation 3.24), P (kg·m<sup>-1</sup>-s<sup>-2</sup>) is fluid pressure,  $\alpha_T$  (K<sup>-1</sup> or "C") is the coefficient of thermal expansion of the fluid, and T (K or "C) is temperature.

# Compaction vs. Aquathermal Pressure

$$\frac{dV}{V} = -BdP + \alpha_T dT \tag{7.9}$$

Now, to calculate the increase in fluid pressure associated with heating the pore fluid, suppose the pore fluid is heated but cannot expand because the pore space is constant. Constant fluid volume implies that dV/V=0, thus

$$BdP = \alpha_T dT$$
 (7.10)

or

$$dP = \alpha_T \frac{dT}{B} \tag{7.11}$$

# Compaction vs. Aquathermal Pressure

Alternatively, to calculate the increase in fluid pressure that occurs from pore collapse alone, suppose that temperature is held constant, but that the pore space changes. Then dT=0 in equation 7.9, and

$$\frac{dV}{V} = -BdP (7.12$$

or

$$dP = \frac{-\frac{dV}{V}}{R}$$
(7.13)

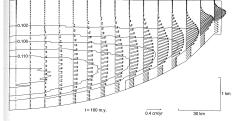
### Compaction vs. Aquathermal Pressure

$$tio = \frac{\alpha_T \Delta T}{\frac{\Delta V}{V}}$$
(7.1)

To evaluate the magnitude of this ratio, consider an average sedimentation rate of 50 m-Ma  $^{-1}$  for MA. Assume that the change of porously of depth may be described by an exponential model,  $\phi = \phi_0^{-1}$ , and take  $\phi_0 = 0.25$  and b = 0.000 m. Let the coefficient of fluid thermal expansion ( $\alpha_1$ ) be  $5 \times 10^{-4}$  Kr. [Choose  $z_1 = 1000$  m depth,  $z_2 = 1050$  m depth, and assume an average prothermal gradient of  $25^{\circ}$ C·K·m<sup>-1</sup>. Then  $\phi_1 = 0.17913$ ,  $\phi_2 = 0.17617$ , and  $\Delta \phi = \phi_1 - \phi_2 = 0.00028$ 0. Define the average protesty as  $\phi = (\phi_1 + \phi_2)^2 = 0.1765$ . We can then approximate  $\Delta VVV$  by  $\Delta \phi_0 \phi = 0.000280$ 0. The Change in temperature from  $z_1$  to  $z_2$  is  $1.25^{\circ}$ C. Thus

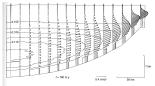
ratio = 
$$\frac{(5 \times 10^{-4})(1.25)}{0.01667} = \frac{1}{27}$$
 (7.16)

## Compaction Driven Flow



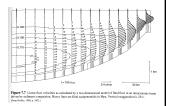
# Compaction Driven Flow

- Fluids expelled from shallow sediments move upwards, while deeper fluids move laterally to basin edges (Figure 7.7).
   Very small excess pressures (exceeding hydrostatic) develop.



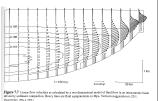
### Compaction Driven Flow

- Fluid velocity scales with sedimentation rate. Maximum linear velocity is about 0.5 cm-yr<sup>-1</sup>; average linear fluid velocity is about 0.2 cm-yr<sup>-1</sup>.
   Compaction-driven fluid flow in intracration basin is not an efficient mechanism for heat and mass transport
   Interlayered sands and shales lead to a very high basin-scale anisotropy, k<sub>f</sub>/k<sub>c</sub> ~ 1000.



## Compaction Driven Flow

- 6. Expelled pore fluids tend to move vertically out of compacting shales (aquitards) into neighboring sandstone aquifers and then travel horizontally to basin edges. (Figure 7.7).
  7. Some pore fluids move downwards (inspect the isopotentials in Figure 7.7). Near the center of the basin, the path of least resistance may be for pore fluids to move downwards to a basal aquifer and then laterally to the basin edge.
  8. In stark contrast to topographically driven flow, flow velocities are not dependent upon permeability so long as the actual compaction process itself is not inhibited or retarded.



# Compaction Driven Flow

