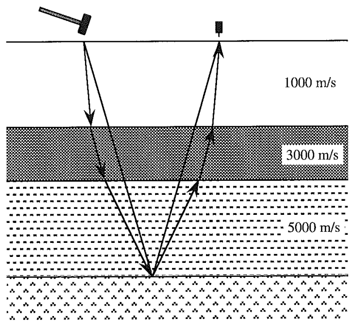
Multiple Layers: Green's Method



Multiple Layers: Dix Equation

The Dix Equation

A second option was derived by C. Hewitt Dix (1955). Dix demonstrated that in a case where there are n horizontal beds, travel-times can be related to actual paths traversed by employing a special velocity value known as the root-mean-square velocity V_{rms}. In the general case where there are n horizontal beds and Δt_i is the *one-way vertical travel-time* through bed i, the Dix equation states that

$$V_{rms}^2 \approx \frac{\sum_{i=1}^{n} V_i^2 \Delta t_i}{\sum_{i=1}^{n} \Delta t_i} \quad (4\text{-}8)$$

For example, if we want to determine the rms velocity to the second in a series of reflecting interfaces, we expand Eq. 4-8 to arrive at

$$V_{rms} = \left(\frac{V_2^2 \Delta t_2 + V_1^2 \Delta t_1}{\Delta t_2 + \Delta t_1} \right)^{1/2} \quad (4\text{-}9)$$

Careful inspection of Eq. 4-9 should lead to an understanding of why this velocity is termed the root-mean-square velocity.

Although we soon will see that the Dix equation provides very good results, note that it is an approximation and assumes that source-receiver distances are kept small relative to the distances to reflecting interfaces.

Dix Equation: Velocities

$$t^2 = \frac{1}{V_{rms}^2} x^2 + t_0^2 \quad (4\text{-}10)$$

We begin with Eq. 4-8. Given n reflecting horizons, let V_{rms_n} represent the rms velocity to the nth reflector and let $V_{rms_{n-1}}$.represent the rms velocity to the (n–1)st reflector. Then

$$V_{rms_n}^2 = \frac{\sum_{i=1}^{n} V_i^2 \Delta t_i}{\sum_{i=1}^{n} \Delta t_i} \quad (4\text{-}11)$$

and

$$V_{rms_n}^2 \sum_{i=1}^{n} \Delta t_i = \sum_{i=1}^{n} V_i^2 \Delta t_i \quad (4\text{-}12)$$

Next we expand the summation on the right of the equal sign in Eq. 4-12 and write an equation for $V_{rms_{n-1}}$.

$$V_{rms_n}^2 \sum_{i=1}^{n} \Delta t_i = \sum_{i=1}^{n-1} V_i^2 \Delta t_i + V_n^2 \Delta t_n \quad (4\text{-}13)$$

$$V_{rms_{n-1}}^2 = \frac{\sum_{i=1}^{n-1} V_i^2 \Delta t_i}{\sum_{i=1}^{n-1} \Delta t_i} \quad (4\text{-}14)$$

$$V_{\text{rms}_{n-1}}^2 \sum_{i=1}^{n-1} \Delta t_i = \sum_{i=1}^{n-1} V_i^2 \Delta t_i \qquad (4\text{-}15)$$

$$V_n^2 = \frac{V_{\text{rms}_n}^2 \sum_{i=1}^{n} \Delta t_i - V_{\text{rms}_{n-1}}^2 \sum_{i=1}^{n-1} \Delta t_i}{\Delta t_n} \qquad (4\text{-}16)$$

All that is left is to recast Eq. 4-16 into a form that uses quantities directly obtainable from an x^2 - t^2 diagram. Remembering that Δt_i is the *one-way vertical travel-time* through bed i and that t_{0_n} is the two-way vertical travel time from reflector n (at $x^2 = 0$ on an x^2 - t_2 diagram), we can write

$$\begin{aligned} \sum_{i=1}^{n} \Delta t_i &= \Delta t_1 + \Delta t_2 + \Delta t_3 \ldots \\ &= \frac{t_{0_1} + \left(t_{0_2} - t_{0_1}\right) + \left(t_{0_3} - t_{0_2}\right)\ldots}{2} \qquad (4\text{-}17) \\ &= \frac{t_{0_n}}{2} \end{aligned}$$

and also,

$$\Delta t_n = \frac{t_{0_n} - t_{0_{n-1}}}{2} \qquad (4\text{-}18)$$

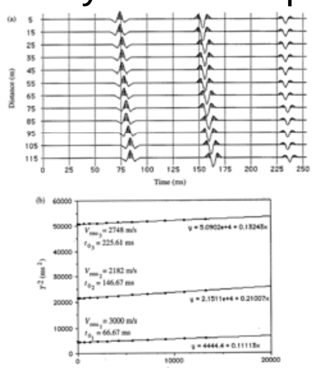
Therefore, we finally arrive at,

$$V_n^2 = \frac{V_{\text{rms}_n}^2 \frac{t_{0_n}}{2} - V_{\text{rms}_{n-1}}^2 \frac{t_{0_{n-1}}}{2}}{\frac{\left(t_{0_n} - t_{0_{n-1}}\right)}{2}} \qquad (4\text{-}19)$$

or

$$V_n^2 = \frac{V_{\text{rms}_n}^2 t_{0_n} - V_{\text{rms}_{n-1}}^2 t_{0_{n-1}}}{t_{0_n} - t_{0_{n-1}}} \qquad (4\text{-}20)$$

Multiple Layers: Dix Equation



Example

$$V_3^2 = \frac{V_{\text{rms}_3}^2 t_{0_3} - V_{\text{rms}_2}^2 t_{0_2}}{t_{0_3} - t_{0_2}}$$

$$V_3^2 = \frac{(2748 \ \text{m/s})^2 \ (0.22561 \ \text{s}) - (2182 \ \text{m/s})^2 (0.14667 \ \text{s})}{(0.22561 \ \text{s} - 0.14667 \ \text{s})}$$

and

$$V_3 = 3569 \ \text{m/s}$$

In many presentations of determining interval velocities the subscripts L for *lower* and U for *upper* are substituted for n and n - 1. Here lower is synonymous with deeper, and upper is synonymous with shallower. Equation 4-20 then becomes

$$V_L^2 = \frac{V_{\text{rms}_L}^2 t_{0_L} - V_{\text{rms}_U}^2 t_{0_U}}{t_{0_L} - t_{0_U}} \qquad (4\text{-}21)$$

Determine Thickness

Once we have interval velocities in hand, calculating thicknesses is trivial. Because travel time equals path distance divided by velocity, vertical thickness (h_n) is equal to velocity times the one-way vertical travel time, or

$$h_n = V_n\left(\frac{t_{0_n} - t_{0_{n-1}}}{2}\right) \qquad (4\text{-}22)$$

If we use this equation to find the thickness of the third unit in our three-unit sequence of Figure 4-13, we have

$$h_3 = V_3\left(\frac{t_{0_3} - t_{0_2}}{2}\right)$$

and

$$h_3 = (3569 \text{ m/s})\left(\frac{0.22561 \text{ s} - 0.14667 \text{ s}}{2}\right) = 141 \text{ m}$$

More Dix Method

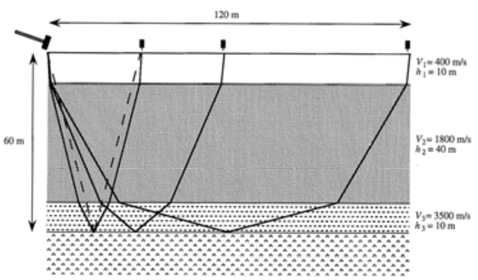


Figure 4-14 Reflection ray paths for source-receiver offsets of 30 m, 60 m, and 120 m for the model parameters noted in the diagram. A straight-line raypath is shown for the 30-m offset.

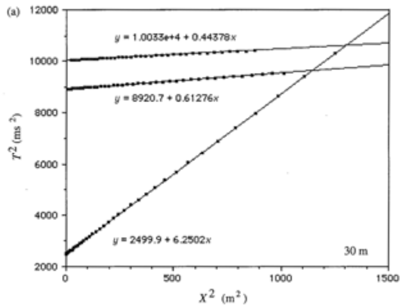
More Dix Method

TABLE 4-4 Reflection Time-Distance Values for Refracted Ray Paths in a Multilayered Sequence

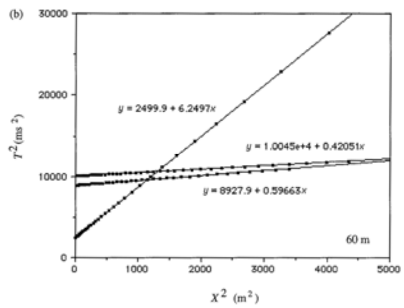
Increment (m)	**0.40**	Velocity 1 (m/s)	**400**	Velocity 2 (m/s)	**1800**	Velocity 3 (m/s)	**3500**
		Thickness 1 (m)	**10**	Thickness 2 (m)	**40**	Thickness 3 (m)	**10**

Theta 1 (°)	X1R (m)	T1R (ms)	Theta 2 (°)	X2R (m)	T2R (ms)	Theta 3 (°)	X3R (m)	T3R (ms)
0.50	0.17	50.00	2.25	3.32	94.48	4.38	4.85	100.21
0.90	0.31	50.01	4.05	5.98	94.56	7.90	8.76	100.33
1.30	0.45	50.01	5.86	8.66	94.69	11.45	12.71	100.52
1.70	0.59	50.02	7.67	11.37	94.87	15.04	16.74	100.78
2.10	0.73	50.03	9.49	14.11	95.09	18.70	20.87	101.13
2.50	0.87	50.05	11.32	16.88	95.37	22.43	25.14	101.56
2.90	1.01	50.06	13.16	19.71	95.71	26.27	29.59	102.08
3.30	1.15	50.08	15.01	22.60	96.10	30.24	34.26	102.71
3.70	1.29	50.10	16.88	25.57	96.55	34.37	39.25	103.47
4.10	1.43	50.13	18.76	28.61	97.07	38.72	44.65	104.39
4.50	1.57	50.15	20.67	31.76	97.66	43.34	50.63	105.51
4.90	1.71	50.18	22.60	35.02	98.32	48.35	57.51	106.92
5.30	1.85	50.21	24.56	38.41	99.08	53.91	65.85	108.78
5.70	2.00	50.25	26.54	41.96	99.93	60.33	77.06	111.47
6.10	2.14	50.28	28.56	45.68	100.89	68.38	96.14	116.39
*********	*********	*********	*********	*********	*********	*********	*********	*********

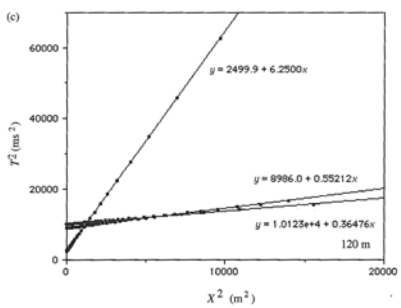
30 m array



60 m array



120 m array

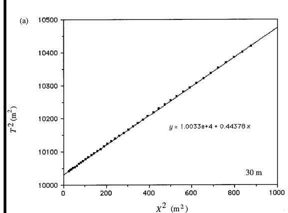


Errors in Green & Dix Solutions

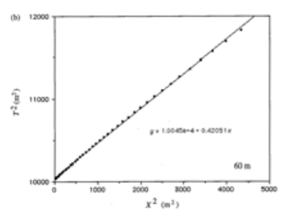
TABLE 4-5 Comparison of Model Parameters with Green and Dix Solutions for Three Geophone Spreads

Parameters	Model	Green 30 m	Green 60 m	Green 120 m	Dix 30 m	Dix 60 m	Dix 120 m
Velocity 1 (m/s)	400	400	400	400	400	400	400
Velocity 2 (m/s)	1800	2250	2293	2411	1812	1839	1912
Velocity 3 (m/s)	3500	5254	5585	6529	3542	3736	4234
Thickness 1 (m)	10	10	10	10	10	10	10
Thickness 2 (m)	40	50	51	54	40	41	43
Thickness 3 (m)	10	15	16	19	10	11	12

30 and 60 m Spreads

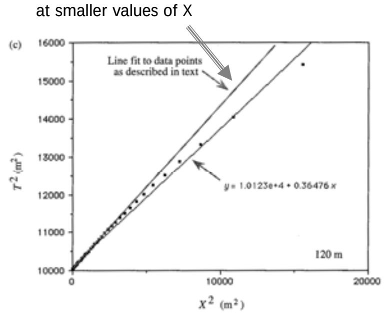


Errors at larger X values
for deeper interfaces



120 m Spread

More weight on values
at smaller values of X



Field Seismogram

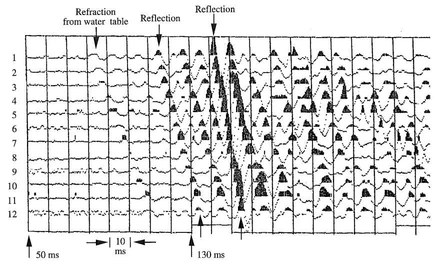


Figure 4-17 Field seismogram from the Connecticut Valley, Massachusetts. Geophone traces are labeled 1–12. The first geophone is located 60 m from the energy source. The geophone interval is 3 m. Reflections selected for analysis are marked by arrows. Timing lines are at 10-ms intervals. The record encompasses 200 ms. Note that the record begins at 50 ms.

X^2-t^2 plot of field seismogram

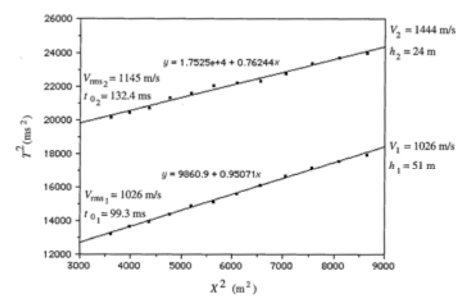


Figure 4-18 An x^2-t^2 plot and results of Dix analysis for reflections identified in Figure 4-17.

Dipping Layer: Travel-time Eqn.

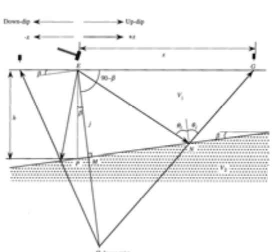


Figure 4-19 Diagram illustrating symbols used in derivation of a travel-time equation for a reflected ray. Dip convention is similar for that used in refraction analysis. β is positive as illustrated.

Path distance is most easily derived by using the law of cosines:

$$a^2 = b^2 + c^2 - 2bc\ (\cos A) \quad (4\text{-}23)$$

where a, b, and c are the sides of the triangle and A is the angle opposite side a. Thus, for the triangle in Figure 4-19,

$$(QG)^2 = (EQ)^2 + (EG)^2 - 2(EQ)(EG)\cos(90-\beta) \quad (4\text{-}24)$$

$$(QG)^2 = (2j)^2 + (x)^2 - 2(2j)(x)\cos(90-\beta)$$

$$(QG)^2 = 4j^2 + x^2 - (4jx)\cos(90-\beta)$$

and

$$(QG)^2 = 4j^2 + x^2 - (4jx)\sin\beta \quad (4\text{-}25)$$

The travel-time equation thus becomes

$$t^2 = \frac{4j^2 + x^2 - 4jx\sin\beta}{V^2} \quad (4\text{-}26)$$

or

$$t = \frac{(4j^2 + x^2 - 4jx\sin\beta)^{1/2}}{V} \quad (4\text{-}27)$$

Split Spread Array –Asymmetric

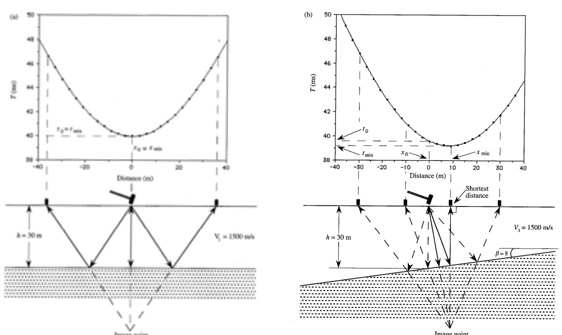
TABLE 4-6 Reflection Time-Distance Values for a Dipping Interface Using a Split-Spread Arrangement

Geophone	Distance (m)	Time (ms)
12	−36	49.09
11	−33	47.91
10	−30	46.79
9	−27	45.73
8	−24	44.74
7	−21	43.81
6	−18	42.96
5	−15	42.18
4	−12	41.49
3	−9	40.88
2	−6	40.36
1	−3	39.94
Source	0	39.61
1	3	39.38
2	6	39.25
3	9	39.23
4	12	39.30
5	15	39.48
6	18	39.76
7	21	40.13
8	24	40.60
9	27	41.17
10	30	41.82
11	33	42.55
12	36	43.36

Velocity (m/s)	**1500**
Thickness (m)	**30**
Dip (°)	**8**
Increment (x) (m)	**3**

t-zero (ms)	39.61
t-min (ms)	39.23
x-min (m)	8.27

Horizontal versus Dipping



Dip, Thickness, Velocity

If we differentiate Eq. 4-26 and set the result equal to zero, we can find the value of x (x_{min}) for which the time term is a minimum. Accordingly,

$$\frac{dV^2t^2}{dx} = 2tV^2\frac{dt}{dx} = 2x - 4j\sin\beta$$

$$\frac{dt}{dx} = \frac{x - 2j\sin\beta}{tV^2} = 0 \qquad \text{(4-28)}$$

and

$$x_{min} = 2j\sin\beta \qquad \text{(4-29)}$$

We now can find an expression for t_{min} by substituting Eq. 4-29 into Eq. 4-26.

$$t_{min}^2 = \frac{4j^2 + (2j\sin\beta)^2 - 4j(2j\sin\beta)\sin\beta}{V^2}$$

$$t_{min}^2 = \frac{4j^2\cos^2\beta}{V^2} \qquad \text{(4-30)}$$

$$t_{min} = \frac{2j\cos\beta}{V} \qquad \text{(4-31)}$$

Dip, Thickness, Velocity

Next we derive an expression for t_0, which, since $x = 0$ at t_0, also is easily produced from Eq. 4-26.

$$t_0 = \frac{2j}{V} \quad (4\text{-}32)$$

If we have good data, we should be able to acquire values for t_0 and t_{min} from a travel-time curve. Realizing this and examining Eqs. 4-31 and 4-32, it follows that

$$\frac{t_{min}}{t_0} = \frac{\frac{2j \cos \beta}{V}}{\frac{2j}{V}} \quad (4\text{-}33)$$

$$\frac{t_{min}}{t_0} = \cos \beta \quad (4\text{-}34)$$

Dip, Thickness, Velocity

or

$$\beta = \cos^{-1}\left(\frac{t_{min}}{t_0}\right) \quad (4\text{-}35)$$

Based on this derivation, the sign of β must be determined from the dip direction, which should be clearly indicated on the travel-time curve.

Because we now know β and can determine x_{min} in the same fashion we obtained t_{min}, Eq 4-29 supplies j.

$$j = \frac{x_{min}}{2 \sin \beta} \quad (4\text{-}36)$$

Finally, we want to determine thickness, h. From Figure 4-19 it follows that

$$h = \frac{j}{\cos \beta} \quad (4\text{-}37)$$

At this point we have values for h, j, β, t_{min}, t_0, and x_{min}. Any of several of the preceding equations can be used to determine V. The template in dynamic Table 4-7 labeled t-min/t-0 takes care of the computational tasks once the appropriate values are obtained from the travel-time curve.

Dip, Thickness, Velocity

Because the line we produced in Figure 4-21 appears to bisect time values for equivalent distances from the source, let's investigate the result of taking an average by summing our travel-time Eq. 4-26 for an inclined interface and substituting $+x$- and $-x$-values.

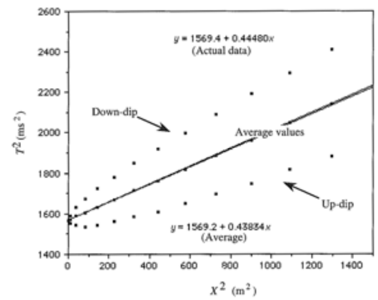
$$t^2 + t^2 = \frac{4j^2 + (-x)^2 - 4j(-x)\sin\beta}{V^2} + \frac{4j^2 + (+x)^2 - 4j(+x)\sin\beta}{V^2} \quad (4\text{-}38)$$

$$2t^2 = \frac{x^2 + 4j^2 + x^2 + 4j^2}{V^2}$$

$$t^2 = \frac{x^2 + 4j^2}{V^2} \quad (4\text{-}39)$$

Equation 4-39 is the same as Eq. 4-6, with the exception that j replaces h, which tells us that an average of our time values at equivalent distances from the source can be used to determine velocity and the value of j.

Plot of Table 4.6 Data



Return to NMO

Recall that the definition of NMO is the difference in reflection travel-times from a horizontal reflecting surface due to variations in the source-geophone distance, or

$$T_{NMO} = t_x - t_0$$

$$T_{NMO} = \frac{\left(x^2 + 4h_1^2\right)^{1/2}}{V_1} - \frac{2h_1}{V_1} \quad (4\text{-}5)$$

Now let's examine our basic travel-time equation once again so that we can express as much as possible in terms of t_0. For convenience in the development we will represent V_1 by V and h_1 by h.

$$t_x = \frac{\left(x^2 + 4h^2\right)^{1/2}}{V} \quad (4\text{-}1)$$

$$t_x^2 = \frac{x^2}{V^2} + \frac{4h^2}{V^2} = \frac{x^2}{V^2} + t_0^2 \quad (4\text{-}40)$$

$$t_x^2 = \frac{x^2 + V^2 t_0^2}{V^2} = \left(\frac{x^2}{V^2 t_0^2} + 1\right) t_0^2 \quad (4\text{-}41)$$

$$t_x = t_0 \left(1 + \frac{x^2}{V^2 t_0^2}\right)^{1/2} \quad (4\text{-}42)$$

Return to NMO

According to the generalized binomial theorem,

$$(1 + z)^a = 1 + az + \frac{a(a-1)}{2 \cdot 1} z^2 + \frac{a(a-1)\cdots(a-n+1)}{n!} z^n + \cdots$$

So, if we set

$$a = \frac{1}{2} \quad \text{and} \quad z = \left(\frac{x^2}{V^2 t_0^2}\right)^{1/2}$$

Eq. 4-42 can be expressed as

$$t_x = t_0 \left(1 + \frac{x^2}{2V^2 t_0^2} - \frac{x^4}{8V^4 t_0^4} + \frac{x^6}{16V^6 t_0^6} + \cdots\right) \quad (4\text{-}43)$$

Examining the variables within the parentheses, we see that we can ignore all but the first two terms, *if*

$$\frac{x}{V t_0} <<< 1$$

If we do so, then our expression reduces to

$$t_x = t_0 + \frac{x^2}{2 t_0 V^2} \quad (4\text{-}44)$$

Return to NMO

Recalling our objective to express T_{NMO} in terms of t_0, we see that since

$$T_{NMO} = t_x - t_0$$

then

$$T_{NMO} = \frac{x^2}{2t_0V^2} \qquad (4\text{-}45)$$

However, we must keep in mind the conditions under which Eq. 4-45 is valid. Equation 4-7 tells us that our former qualification

$$\frac{x}{Vt_0} <<< 1$$

is equivalent to saying

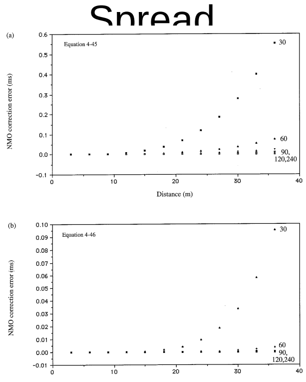
$$\frac{x}{2h} <<< 1$$

Thus, for Eq. 4-44 to be a useful approximation, horizontal distance divided by twice the thickness must be much less than one.

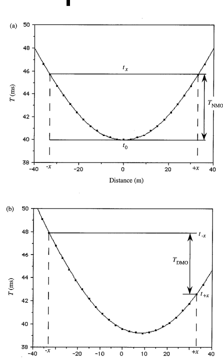
$$T_{NMO} = \frac{x^2}{2t_0V^2} - \frac{x^4}{8t_0{}^3V^4} \qquad (4\text{-}46)$$

$$T_{NMO} = \frac{x^2}{2t_0V_{rms}^2} \qquad (4\text{-}47)$$

NMO Approx: 30-240 m Spread



Dip Normal Moveout (DMO)



Dip move-out (DMO) is the difference in travel-time to geophones at equal distances from the source position ($x = 0$) in a split-spread geometry, or

$$T_{DMO} = t_{+x} - t_{-x} \qquad (4\text{-}48)$$

DMO

$$t_x = \frac{\left(4j^2 + x^2 - 4jx \ \sin\beta\right)^{1/2}}{V} \qquad (4\text{-}27)$$

$$t_x^{\ 2} = \frac{x^2}{V^2} + \frac{4j^2}{V^2} - \frac{4jx \sin\beta}{V^2} \qquad (4\text{-}49)$$

$$t_x^{\ 2} = t_0^{\ 2} + \frac{x^2}{V^2} - \frac{4jx \sin\ \beta}{V^2} \qquad (4\text{-}50)$$

$$t_x^{\ 2} = t_0^{\ 2}\left(1 + \frac{x^2 - 4jx \sin\ \beta}{4j^2}\right) \qquad (4\text{-}51)$$

and

$$t_x = t_0\left(1 + \frac{x^2 - 4jx \ \sin\beta}{4j^2}\right)^{1/2} \qquad (4\text{-}52)$$

Expanding Eq. 4-52, we have

$$t_x = t_0\left[1 + \left(\frac{x^2 - 4jx \ \sin\beta}{8j^2}\right) - \left(\frac{x^2 - 4jx \ \sin\beta}{32j^2}\right)^2 + \cdots\right] \qquad (4\text{-}53)$$

Once again, we take only the first two terms in the expansion, and, remembering that $t_0 = 2j/V$, we arrive at

$$t_x = t_0 + \frac{x^2 - 4jx \ \sin\ \beta}{4jV} \qquad (4\text{-}54)$$

DMO

When we developed the x^2-t^2 *average data* method we summed reflection travel-time equations for $+x$ and $-x$ cases. However, recalling that we are developing a method to take advantage of the availability of dip move-out which we defined in Eq. 4-48, we see that

$$T_{DMO} = t_{+x} - t_{-x} = \left(t_0 + \frac{(+x)^2 - 4j(+x)\sin\ \beta}{4jV}\right) - \left(t_0 + \frac{(-x)^2 - 4j(-x)\sin\ \beta}{4jV}\right) \qquad (4\text{-}55)$$

$$T_{DMO} = -\frac{2x\sin\beta}{V} \qquad (4\text{-}56)$$

and

$$\beta = \sin^{-1}\left(-\frac{VT_{DMO}}{2x}\right) \qquad (4\text{-}57)$$

We determine T_{DMO} for a selected x-value on our travel-time curve and V from an x^2 - t^2 plot. This gives us β from Eq. 4-57 and j, and then h, from one of several equations presented previously.