Multiple Layers: Green’s Method

\[ v_\text{g} = \frac{\sum v_i b_i}{\sum b_i} \]  

1000 m/s

Multiple Layers: Dix Equation

The Dix Equation

A second option was derived by C. H. Dix (1957). Dix demonstrated that in a case where there are a horizontal beds, traveltimes can be related to actual path traversed if we employ a special velocity value known as the root-mean-square velocity \( v_{rm} \). In the general case where there are a horizontal beds and \( \Delta r \) is the one-way vertical interval through bed, the Dix equation states that

\[ v_{rm} = \sqrt{\frac{\sum v_i^2 b_i}{\sum b_i}} \]  

For example, if we want to determine the max velocity to the second in a series of reflecting interfaces, we expand Eq. 4.8 to arrive at

\[ v_{rm} = \left( \frac{\sum v_i b_i}{\sum b_i} + \frac{v_f b_f}{b_f} \right)^{1/2} \]  

Careful inspection of Eq. 4.9 should lead to an understanding of why this velocity is termed the root-mean-square velocity.

Although we saw earlier that the Dix equation provides very good results, note that it is an approximation and assumes that source-receiver distances are kept small relative to the distance to reflecting interfaces.

Dix Equation: Velocities

\[ v^2 = \frac{1}{\delta^2} v_\text{g}^2 + v_f^2 \]  

We begin with Eq. 4.6. Given a reflecting horizon, let \( v_{\text{rme}} \) represent the rms velocity to the nth reflector and let \( v_{\text{rme}} \) represent the rms velocity to the (n-1)st reflector. Then

\[ v_{\text{rme}} = \frac{\sum v_i b_i}{\sum b_i} \]  

and

\[ v_{\text{rme}} = \frac{\sum v_i b_i}{\sum b_i} \]  

Next we expand the summation on the right of the equal sign in Eq. 4.12 and write an equation for \( v_{\text{rme}} \),

\[ v_{\text{rme}} = \frac{\sum v_i b_i}{\sum b_i} \]
\[
\begin{align*}
\bar{V}_k &= \frac{\int_{s_0}^{s_{k-1}} \overline{V}_k \, ds}{s_{k-1} - s_0} \\
\bar{V} &= \frac{\int_{s_0}^{s_{k-1}} \bar{V} \, ds}{s_{k-1} - s_0}
\end{align*}
\]

All that is left is to recall Eq. 4.16 from a form that non-quantities directly observable from an \( s' \) = 0 condition. Denoting that \( \bar{V}_k \) is the one-way velocity recorded from oil reference \( s' = 0 \) on \( s' = s \) by \( \bar{V} \), we can write:

\[
\bar{V}_k = \bar{V}_k + \bar{V} + \frac{\bar{V} \Delta s_0}{2}
\]

and also:

\[
\bar{V}_k = \frac{\bar{V}_k - \bar{V}}{2}
\]

Therefore, we divide them as:

\[
\bar{V}_k = \frac{\bar{V}_k - \bar{V}}{2} \quad \text{(4.13)}
\]

or

\[
\bar{V}_k = \frac{\bar{V}_k - \bar{V}}{2} \quad \text{(4.14)}
\]

Multiple Layers: Dix Equation

Example

\[
\begin{align*}
\overline{V}_1 &= \frac{V_{12}^2 - V_{10}^2}{V_{12} - V_{10}} \\
\overline{V}_1 &= \frac{[2740 \text{ m/s}]^2 - [2182 \text{ m/s}]^2}{0.2250 \text{ s} - 0.1466 \text{ s}}
\end{align*}
\]

and

\[
\overline{V}_1 = 3510 \text{ m/s}
\]

In many presentations of determining interval velocities the subscripts 1 for lower and 0 for upper are substituted for \( s' \) and \( s' = 0 \). Then lower is synonymous with deeper, and upper is synonymous with shallower. Equation 4.20 then becomes

\[
\overline{V}_1 = \frac{V_{12}^2 - V_{10}^2}{V_{12} - V_{10}} \quad \text{(4.21)}
\]
Determine Thickness

Once we have known velocities in hand, calculating thicknesses is trivial. Because travel time equals path distance divided by velocity, vertical thickness ($h_v$) is equal to velocity times the one-way travel time, or

$$h_v = V_v \frac{S_v - S_{v0}}{2}$$

If we use this equation to find the thickness of the third unit in our three-unit sequence of Figure 9.12, we have

$$h_v = V_v \sqrt{\frac{S_v - S_{v0}}{2}}$$

and

$$h_3 = (3569 \text{ m/s}) \left( \frac{0.022563 - 0.14667}{2} \right) = 141 \text{ m}$$

More Dix Method

Figure 6.41: Refraction rays for assumed source offsets of 30 m, 40 m, and 120 m for the model parameters used in the diagram. Arrows less represent ray paths for the 30 m offset.

More Dix Method

| Table 6.4: Refraction Time-Distance Values for Refracted Ray Paths in a Multilayered Sequence |
|---|---|---|---|---|---|---|---|---|---|
| Distance (m) | 0.00 | 10.00 | 20.00 | 30.00 | 40.00 | 50.00 | 60.00 | 70.00 | 80.00 |
| Thickness 1 (m) | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 |
| Velocity 1 (m/s) | 50.00 | 50.01 | 50.02 | 50.03 | 50.04 | 50.05 | 50.06 | 50.07 | 50.08 |
| Time 1 (s) | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 |
| Time 2 (s) | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 |
| Time 3 (s) | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 |
| Thickness 2 (m) | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 |
| Velocity 2 (m/s) | 50.00 | 50.01 | 50.02 | 50.03 | 50.04 | 50.05 | 50.06 | 50.07 | 50.08 |
| Time 4 (s) | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 |
| Time 5 (s) | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 |
| Time 6 (s) | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 |
| Thickness 3 (m) | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 |
| Velocity 3 (m/s) | 50.00 | 50.01 | 50.02 | 50.03 | 50.04 | 50.05 | 50.06 | 50.07 | 50.08 |
| Time 7 (s) | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 |
| Time 8 (s) | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 |
| Time 9 (s) | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 |
| Time 10 (s) | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 |

---

3
Errors in Green & Dix Solutions

<table>
<thead>
<tr>
<th>TABLE 6-5: Comparison of Model Parameters with Green and Dix Solutions for Three Geophone Spreads</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Velocity 1 (m/s)</td>
</tr>
<tr>
<td>Velocity 2 (m/s)</td>
</tr>
<tr>
<td>Velocity 3 (m/s)</td>
</tr>
<tr>
<td>Thickness 1 (m)</td>
</tr>
<tr>
<td>Thickness 2 (m)</td>
</tr>
<tr>
<td>Thickness 3 (m)</td>
</tr>
</tbody>
</table>

30 and 60 m Spreads

Errors at larger X values for deeper interfaces

120 m Spread

More weight on values at smaller values of X
Field Seismogram

X²-t² plot of field seismogram

Dipping Layer: Travel-time Eqn.
Split Spread Array – Asymmetric

TABLE 4-4: Reflective Time-Distance Values for a Dipping Interface Using a Split Spread Arrangement

<table>
<thead>
<tr>
<th>Channel</th>
<th>Distance (m)</th>
<th>Time (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>47.10</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>46.79</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>47.72</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>47.74</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
<td>47.74</td>
</tr>
<tr>
<td>6</td>
<td>60</td>
<td>47.74</td>
</tr>
<tr>
<td>7</td>
<td>70</td>
<td>47.74</td>
</tr>
<tr>
<td>8</td>
<td>80</td>
<td>47.74</td>
</tr>
<tr>
<td>9</td>
<td>90</td>
<td>47.74</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>47.74</td>
</tr>
</tbody>
</table>

Horizontal versus Dipping

Dip, Thickness, Velocity

If we differentiate Eq. 4-26 and set the result equal to zero, we can find the value of $x_{(s_{max})}$ for which the time term is a minimum. Accordingly,

$$\frac{d^2 T}{dx^2} = 2x_1 \frac{d^2 x_1}{dx^2} = 2x - 4x \sin \beta$$

and

$$\frac{dx_1}{dx} = -\frac{3}{2} x \sin \beta$$

(4-28)

We now can find an expression for $t_{(s_{max})}$ by substituting Eq. 4-29 into Eq. 4-26.

$$t_{(s_{max})} = \frac{4x_1}{V} \left( 1 - \sin \beta \right) - \frac{4x_1}{V} \left( 1 - \sin \beta \right) \sin \beta$$

$$t_{(s_{max})} = \frac{4x_1}{V} \cos \beta \sin \beta$$

(4-30)

$$t_{(s_{max})} = \frac{3x_1}{V} \sin \beta$$

(4-31)
Dip, Thickness, Velocity

Next we derive an expression for \( \psi \), which, since \( z = 0 \) at \( \psi \), also is easily produced from Eq. 4-36.

\[
\psi = \frac{2 \cos \beta}{\cos \beta}
\]

(4-32)

If we have good data, we should be able to acquire values for \( \psi \) and \( \chi_{\psi} \) from a travel-time curve. Realizing this and examining Eqs. 4-31 and 4-32, it follows that

\[
\tan \psi = \frac{2 \cos \beta}{3 \sin \beta}
\]

(4-33)

\[
\cot \psi = \cos \beta
\]

(4-34)

Dip, Thickness, Velocity

or

\[
\beta = \cos^{-1} \left( \frac{\tan \psi}{3 \sin \beta} \right)
\]

(4-35)

Based on this derivation, the sign of \( \beta \) must be determined from the dip direction, which should be clearly indicated on the travel-time curve.

Because we now have \( \beta \) and can determine \( \tan \psi \) in the same fashion we obtained \( \chi_{\psi} \), Eq. 4-29 supplies

\[
\psi = \frac{2 \cos \beta}{3 \sin \beta}
\]

(4-36)

Finally, we want to determine thickness, \( h \). From Figure 4-19 it follows that

\[
h = \frac{\chi_{\psi}}{\psi}
\]

(4-37)

At this point we have values for \( h, \beta, \chi_{\psi}, \psi \) and \( \chi_{\psi} \). Any of several of the preceding equations can be used to determine \( \psi \). The technique in Table 4-1 (labeled result 0) takes care of the computational tasks once the appropriate values are obtained from the travel-time curve.

Dip, Thickness, Velocity

Because the line we produced in Figure 4.21 appears to be a time value for equivalent distances from the source. Let’s investigate the result of taking an average by using our travel-time Eq. 4-31 for an inclined transmitter and substituting \( s \times \) and \( v \times \) values.

\[
\rho^2 = \left( s_1 - s_2 \right)^2 - 4 \left( s_1 \right) \left( s_2 \right) \sin \beta \left( \rho^2 = \left( x_1 - x_2 \right)^2 - 4 \left( x_1 \right) \left( x_2 \right) \sin \beta \right)
\]

(4-38)

\[
2 \rho^2 = \left( s_1 + s_2 \right)^2 + 4 s_1^2 - 4 s_2^2
\]

(4-39)

Equation 4-38 is the same as Eq. 4-4, with the exception that \( f \) replaces \( h \), which tells us that an average of our time values at equivalent distances from the source can be used to determine velocity and the value of \( \beta \).
Plot of Table 4.6 Data

Return to NMO

Recall that the definition of NMO is the difference in reflection travel times from a horizontal reflecting surface due to variations in the source-receiver distance, or

\[ t_{\text{NMO}} = t_i - t_s \]

(4.5)

Now let's examine the basic travel-time equation once again so that we can express as much as possible in terms of \( t_0 \). For convenience in the development we will represent \( V \) by \( V \) and \( h \) by \( h \).

\[ t_i = \left( \frac{x^2 + 4dh}{2V^2} \right)^{1/2} \]  

(4.31)

\[ t_i = \frac{x^2}{2V^2} + \frac{dh}{V} \]  

(4.40)

\[ t_i = \frac{x^2}{2V^2} + \frac{h^2}{V^2} \left( \frac{1}{2} \right) \]  

(4.41)

\[ t_i = t_0 \left( 1 + \frac{h^2}{V^2} \right) \]  

(4.42)

According to the generalized Taylor's theorem,

\[ (1 + a)^n = 1 + na + \frac{n(n-1)}{2!}a^2 + \frac{n(n-1)(n-2)}{3!}a^3 + \ldots \]

So, if we set

\[ a = \frac{h^2}{V^2} \]  

\[ t_i = t_0 \left( 1 + \frac{h^2}{V^2} \right) \]  

(4.43)

Examining the variation within the parentheses, we see that we can ignore all but the first two terms, if

\[ \frac{h^2}{V^2} \ll 1 \]

If we do so, then our expression reduces to

\[ t_i = t_0 + \frac{x^2}{2V^2} \]  

(4.44)
Return to NMO

Recalling our objective to express $T_{x_{0x}}$ in terms of $t_0$, we see that since

$$T_{x_{0x}} = t_x - t_0$$

then

$$T_{x_{0x}} = \frac{x^2}{2V_0^2}$$

(4-45)

However, we must keep in mind the conditions under which Eq. 4-45 is valid. Equation 4-45 is only valid when $x/2V_0 \ll 1$

Now, if $2V_0 > x$, it is equivalent to saying $x/2V_0 < 1$

Thus, for Eq. 4-44 to be a good approximation, horizontal distance divided by velocity must be much less than one.

$$T_{x_{0x}} = \frac{x^2}{4V_0^2}$$

(4-46)

$$T_{x_{0x}} = \frac{x^2}{2V_0^2}$$

(4-47)

---

NMO Approx: 30-240 m

---

Dip Normal Moveout (DMO)

---
DMO

\[
\tau_{s} = \left( \frac{4x_{s}^{2} + x_{s} - \frac{4x_{s} \sin \beta}{4x_{s} \pm \sin \beta}}{\sqrt{2}} \right)
\]  
(4-27)

\[
\tau_{s} = \frac{x_{s}^{2} + \frac{x_{s}}{4} - \frac{2x_{s} \sin \beta}{4x_{s} \pm \sin \beta}}{\sqrt{2}}
\]  
(4-28)

\[
\tau_{s} = \frac{x_{s}^{2} + \frac{x_{s}}{4} - \frac{2x_{s} \sin \beta}{4x_{s} \pm \sin \beta}}{\sqrt{2}}
\]  
(4-29)

\[
\tau_{s} = \frac{\left( \frac{1}{4} \left( \sqrt{2}x_{s}^{2} - \frac{4x_{s} \sin \beta}{4x_{s} \cos \beta} \right) \right)^{1/2}}{\sqrt{2}}
\]  
(4-30)

\[
\tau_{s} = \frac{\left( \frac{1}{4} \left( \sqrt{2}x_{s}^{2} - \frac{4x_{s} \sin \beta}{4x_{s} \cos \beta} \right) \right)^{1/2}}{\sqrt{2}}
\]  
(4-31)

Definition Eq. 4-32, we have:

\[
\tau_{s} = \frac{1}{4} \left[ \left( \frac{1}{4} \left( \sqrt{2}x_{s}^{2} - \frac{4x_{s} \sin \beta}{4x_{s} \cos \beta} \right) \right)^{1/2} - \left( \frac{1}{4} \left( \sqrt{2}x_{s}^{2} - \frac{4x_{s} \sin \beta}{4x_{s} \cos \beta} \right) \right)^{1/2} \right]^{1/2}
\]  
(4-33)

Once again, we take only the first two terms in the expansion, and remembering that \( \tau_{s} = T_{\text{max}} \), we arrive at:

\[
\tau_{s} = \frac{1}{4} \left( \frac{1}{4} \left( \sqrt{2}x_{s}^{2} - \frac{4x_{s} \sin \beta}{4x_{s} \cos \beta} \right) \right)^{1/2}
\]  
(4-34)

DMO

When we developed the \( x^2 \) average axial axis method we summed reflection travel-time equations for both \( x \) cases. However, recalling that we are developing a method to take advantage of the availability of the mean-out which we defined in Eq. 4-46, we see that:

\[
T_{\text{max}} = T_{\text{ave}} - \tau_{s} = \left( \frac{1}{4} \left( \sqrt{2}x_{s}^{2} - \frac{4x_{s} \sin \beta}{4x_{s} \pm \sin \beta} \right) \right) - \left( \frac{1}{4} \left( \sqrt{2}x_{s}^{2} - \frac{4x_{s} \sin \beta}{4x_{s} \cos \beta} \right) \right)
\]  
(4-35)

\[
T_{\text{max}} = -\frac{1}{4} \frac{4x_{s} \sin \beta}{4x_{s} \cos \beta}
\]  
(4-36)

and

\[
\beta = \sin \left( -\frac{T_{\text{max}}}{2x_{s}} \right)
\]  
(4-37)

We determine \( T_{\text{max}} \), for a selected \( x \)-value on our travel-time curve and \( \gamma \) from its \( x^2 \)-plot. This gives us \( \beta \) from Eq. 4-37 and \( x_{s} \) and then \( x_{s} \) from one of several equations presented previously.