Electrical Resistivity

the battery must move positive charges from a low potential at the negative terminal to high potential at the positive terminal. The work done in this potential change requires that a force be applied. This force is known as electromotive force or *emf*. The unit of emf is the *volt* (V).

A 9-V battery maintains a potential difference of 9 V between its terminals and thus has a certain potential for doing work. As noted, the movement of charges through the conducting wire is termed *current*. Specifically,

$$i = \frac{q}{t} \qquad (5\text{-}1)$$

where i is current in amperes, q is charge in coulombs, and t is time in seconds.

Another and very important concept in electrical resistivity surveying is the current density j. Current density is defined as the current divided by the cross-sectional area of the material through which it is flowing,

$$j = \frac{i}{A} \qquad (5\text{-}2)$$

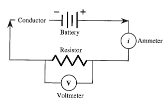


Figure 5-1 A simple electrical circuit illustrating standard symbols for common components.

Current Density

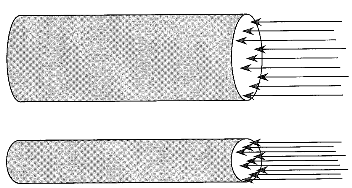


Figure 5-2 Diagram illustrating concept of current density j in wires with different cross-sectional areas. Current flow is represented by arrows.

You recognize, most likely, that copper wire, wood, aluminum, and rubber possess varying resistances to the flow of current. Copper has very low resistance whereas rubber has an extremely high resistance. Resistance is quantified in the following way: 1 *ohm* (Ω) of resistance allows a current of 1 ampere to flow when 1 V of emf is applied.

Ohm's Law

Ohm's law, first presented by German physicist Georg Simon Ohm, states that current is directly proportional to voltage V and inversely proportional to resistance R, or

$$i = \frac{V}{R} \qquad (5\text{-}3)$$

Consider Figure 5-1. If the battery supplies 9 V, and the resistor has a value of 10 Ω, the current measured by the ammeter will be 0.9 amperes. Or, if resistance is increasing, it will take an increasing voltage to maintain the same current.

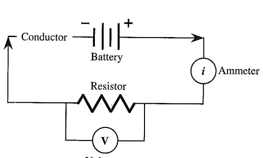


Figure 5-1 A simple electrical circuit illustrating standard symbols for common components.

Resistance and Resistivity

This behavior suggests that the resistances of the resistors in Figure 5-3 depend on their length and cross-sectional areas and also to a fundamental property of the material used in their construction, which we term *resistivity* and denote by ρ. Based on our discussion, we can say that

$$R = \rho \frac{l}{A} \qquad (5\text{-}4)$$

or

$$\rho = R \frac{A}{l} \qquad (5\text{-}5)$$

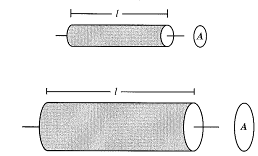


Figure 5-3 Two resistors of different lengths l and different cross-sectional areas A.

Resistance and Resistivity

The resistivity unit is resistance · length, which is commonly denoted by Ω · m. Conductance is the inverse of resistance, and conductivity is the inverse of resistivity.

> Copper has a resistivity or 1.7×10^{-8} Ω • m. What is the resistance of 20 m of copper wire with a cross-sectional radius of 0.005 m? Quartz has a resistivity of 1×10^{16} Ω • m. What is the resistance of a quartz wire of the same dimensions?

R = r(l/A) = 1.7e-8*(20/3.14*(0.005)^2)

CURRENT FLOW IN A HOMOGENEOUS, ISOTROPIC EARTH

Point Current Source

Because the resistivity method consists of applying current and measuring potentials, we begin by considering the potential at a point P_1 when current is applied at a point source C_1. We place the return current electrode at a very great distance and assume material of uniform resistivity ρ. Because air has infinite resistivity, no current flows upward. Thus, current flows radially outward through the earth equally in all directions so as to define a hemispherical surface (Fig. 5-4). Because current distribution is equal everywhere on this surface, which is at a distance r from the current electrode C_1, the potential also is equal. These surfaces are known as *equipotential* surfaces. If we define a very thin shell of thickness dr and employ Eqs. 5-3 and 5-4, we can define the potential difference across the shell to be

$$dV = i(R) = i\left(\rho \frac{l}{A}\right) = i\left(\rho \frac{dr}{2\pi r^2}\right) \qquad (5\text{-}6)$$

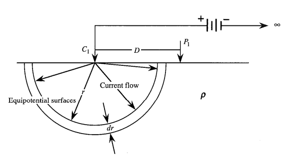


Figure 5-4 Diagram illustrating symbols and configuration used to determine potential at P_1 for a single point source of current C_1. The current sink, C_2, is at infinity. The two equipotential surfaces shown are separated by the distance dr.

Current Flow

We now use Eq. 5-6 to determine the potential at P_1. In determining the potential at a point, we compare it to the potential at a point infinitely far away, which by convention is arbitrarily defined to equal zero. The most direct way to determine V is to integrate Eq. 5-6 over its distance D to the current electrode to infinity, or

$$V = \int_D^\infty dV = \frac{i\rho}{2\pi}\int_D^\infty \frac{dr}{r^2} = \frac{i\rho}{2\pi D} \qquad (5\text{-}7)$$

(Van Nostrand and Cook, 1966, p. 28). Equation 5-7 is the *fundamental* equation

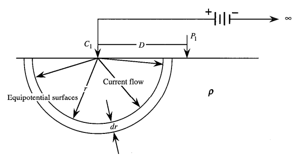


Figure 5-4 Diagram illustrating symbols and configuration used to determine potential at P_1 for a single point source of current C_1. The current sink, C_2, is at infinity. The two equipotential surfaces shown are separated by the distance dr.

Current Flow

The potential at point P_1 is determined by using Eq. 5-7. The effect of the source at C_1 (+) and the sink at C_2 (−) are both considered, and, therefore,

$$V_{P_1} = \frac{i\rho}{2\pi r_1} + \left(-\frac{i\rho}{2\pi r_2}\right) \qquad (5\text{-}8)$$

Expressing r_1 and r_2 in terms of the x-z-coordinate system illustrated in Figure 5-5, we rewrite Eq. 5-8 as

$$V_{P_1} = \frac{i\rho}{2\pi}\left\{\frac{1}{\left[\left(d/2 + x\right)^2 + z^2\right]^{1/2}} - \frac{1}{\left[\left(d/2 - x\right)^2 + z^2\right]^{1/2}}\right\} \qquad (5\text{-}9)$$

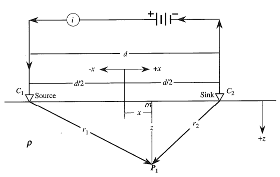


Figure 5-5 Diagram illustrating symbols and configuration used to determine potential at P_1 for a current source C_1 and sink C_2.

Current Penetration is a function of separation of current electrodes

Along a vertical plane midway between the two current electrodes, the fraction of the total current i_f penetrating to depth z for an electrode separation of d is given by

$$i_f = \frac{2}{\pi}\tan^{-1}\left(\frac{2z}{d}\right) \qquad (5\text{-}10)$$

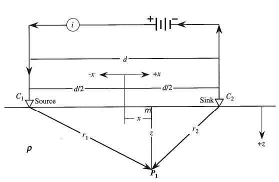


Figure 5-5 Diagram illustrating symbols and configuration used to determine potential at P_1 for a current source C_1 and sink C_2.

Distribution of Potential

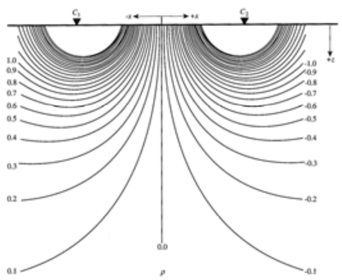


Figure 5-6 Contours based on potential values in Table 5-1. Contours represent positions of equipotential surfaces about a current source C_1 and sink C_2. Many contours near the current electrodes are not shown due to the close spacing of contours in this region.

TABLE 5-2 Percent Current Penetrating a Homogeneous, Isotropic Earth

Depth (m)	Depth/Electrode Separation	% of Total Current
1	0.1	13
2	0.2	24
3	0.3	34
4	0.4	43
5	0.5	50
6	0.6	56
7	0.7	61
8	0.8	64
9	0.9	68
10	1	70
11	1.1	73
12	1.2	75
13	1.3	77
14	1.4	78
15	1.5	80
16	1.6	81
17	1.7	82
18	1.8	83
19	1.9	84
20	2	84
Current electrode separation (m)		**10**

Current Penetration is a function of separation of current electrodes

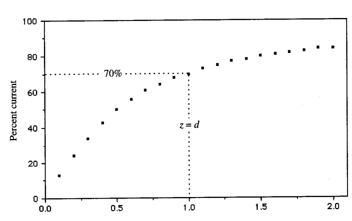


Figure 5-7 Plot of results in Table 5-2. Depth is z and current electrode separation is d. The data points illustrate the extent to which current penetrates into a homogeneous, isotropic Earth.

Lines of Current Flow

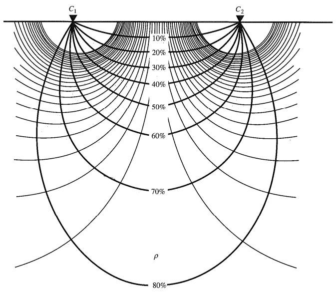


Figure 5-8 Equipotential surfaces and current lines of flow. Labels indicate percent of total current that penetrates to the depth of the line.

Measuring Resistivity

Figure 5-9 illustrates two potential electrodes P_1 and P_2 that are located on the surface as are the current electrodes. Using the equation we have already derived to determine the potential at a point due to a source and a sink, we obtain the potential difference by determining the potential at one potential electrode P_1 and subtracting from it the potential at P_2. Using Eq. 5-8 we determine that

$$V_{P_1} = \frac{i\rho}{2\pi r_1} - \frac{i\rho}{2\pi r_2} \qquad (5\text{-}11)$$

and

$$V_{P_2} = \frac{i\rho}{2\pi r_3} - \frac{i\rho}{2\pi r_4} \qquad (5\text{-}12)$$

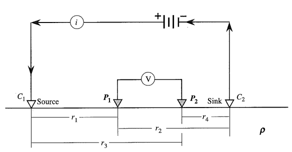


Figure 5-9 Diagram used to determine potential difference at two potential electrodes P_1 and P_2.

Therefore, the potential difference ΔV equals

$$\Delta V = V_{P_1} - V_{P_2} = \left(\frac{i\rho}{2\pi r_1} - \frac{i\rho}{2\pi r_2}\right) - \left(\frac{i\rho}{2\pi r_3} - \frac{i\rho}{2\pi r_4}\right) \qquad (5\text{-}13)$$

or

$$\Delta V = \frac{i\rho}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{r_3} + \frac{1}{r_4}\right) \qquad (5\text{-}14)$$

In the resistivity method, current is entered into the ground, potential difference is measured, and resistivity determined. Because resistivity is the unknown quantity we normally hope to determine, we solve Eq. 5-14 for ρ and obtain

$$\rho = \frac{2\pi\Delta V}{i}\left(\frac{1}{\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{r_3} + \frac{1}{r_4}}\right) \qquad (5\text{-}15)$$

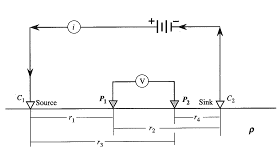


Figure 5-9 Diagram used to determine potential difference at two potential electrodes P_1 and P_2.

Perhaps we should test our understanding of Eq. 5-15 by applying it to a known situation. Let's assume we can place potential electrodes anywhere along the surface, as illustrated in Figure 5-9. Further, we will use the values in Table 5-1 for our test. Figure 5-10(a) presents one possible measurement, and Figure 5-10(b) another. Substituting the values in Figure 5-10(a) produces Eq. 5-16, which results in a resistivity value of 50 Ω · m. A glance at the model values used to produce Table 5-1 confirms that the resistivity is 50 Ω · m.

$$\rho = \frac{2\pi(4.5\text{V})}{1\text{ ampere}}\left(\frac{1}{\frac{1}{3\text{ m}} - \frac{1}{7\text{ m}} - \frac{1}{8\text{ m}} + \frac{1}{2\text{ m}}}\right) = 50\ \Omega\cdot\text{m} \qquad (5\text{-}16)$$

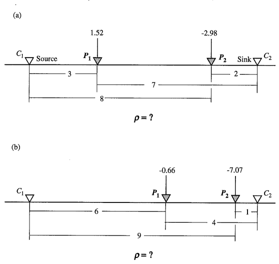


Figure 5-10 Measuring potential difference to determine ρ. Potential values for the indicated spacings are taken from Table 5-1. Solutions are presented in the text (Eqs. 5-16 and 5-17).

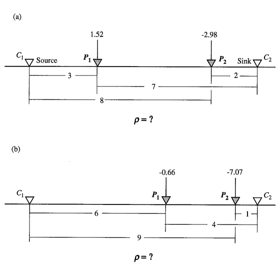


Figure 5-10 Measuring potential difference to determine ρ. Potential values for the indicated spacings are taken from Table 5-1. Solutions are presented in the text (Eqs. 5-16 and 5-17).

Equation 5-17 uses the values illustrated in Figure 5-10(b). As expected, evaluation of the equation gives 50 Ω · m as the resistivity.

$$\rho = \frac{2\pi(6.41\text{V})}{1\text{ ampere}}\left(\frac{1}{\frac{1}{6\text{ m}} - \frac{1}{4\text{ m}} - \frac{1}{9\text{ m}} + \frac{1}{1\text{ m}}}\right) = 50\ \Omega\cdot\text{m} \qquad (5\text{-}17)$$

These calculations confirm that if we produce a current, measure the electrode spacings, and determine the potential difference, we can arrive at a value for the resistivity of the subsurface materials.

Two Layers

Current Distribution

An important goal in this section is to gain a qualitative understanding for the pattern of current distribution in the subsurface when a single horizontal interface separates materials of different resistivities. Our first step toward this goal is to employ an equation that tells us the fraction of the current that penetrates below the interface. This current fraction is given by

$$i_F = \frac{2\rho_1}{\pi\rho_2}(1+k)\sum_{n=0}^{\infty} k^n\left\{\frac{\pi}{2} - \tan^{-1}\left[\frac{2(2n+1)z}{3a}\right]\right\} \qquad (5\text{-}18)$$

where

$$k = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

Two Layers

Equation 5-18 is sufficiently straightforward to permit entry into a spreadsheet for evaluation, and Table 5-3 is the result. The appearance of this table is slightly different from those we've seen previously, as the equation being evaluated is broken down into several components due to its complexity. The major variables in the equation are placed in the upper left-hand corner. The result is given as percent current and is located in the upper right. The components are

$$k = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

$$C = \frac{2\rho_1}{\pi\rho_2}(1 + k)$$

$$k\text{-factor} = k^n$$

$$\text{remainder} = \left\{\frac{\pi}{2} - \tan^{-1}\left[\frac{2(2n + 1)z}{3\,\text{a}}\right]\right\}$$

and

$$\text{result} = k\text{-factor times remainder}$$

TABLE 5-3 Percent Current Penetrating Below a Single Horizontal Interface

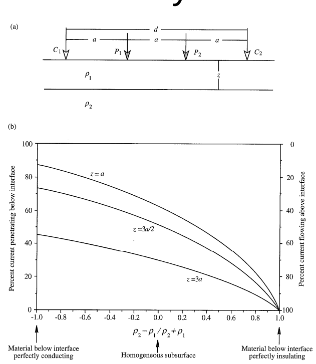
Electrode spacing a in m	**8**
Depth of interface (m)	**8**
ρ_1	**70**
ρ_2	**30**
(Resistivities are in $\Omega \cdot$ m)	

Percent current	74

k	−0.40
C	0.89

n	k-factor	Remainder	Result
0.00	1.00	0.98	0.98
1.00	−0.40	0.46	−0.19
2.00	0.16	0.29	0.05
3.00	−0.06	0.21	−0.01
4.00	0.03	0.17	0.00
5.00	−0.01	0.14	0.00
6.00	0.00	0.11	0.00
7.00	0.00	0.10	0.00
8.00	0.00	0.09	0.00
9.00	0.00	0.08	0.00
10.00	0.00	0.07	0.00
		Sum of results	0.83
		Current fraction	0.74

Two Layers



Refraction of Flow Lines

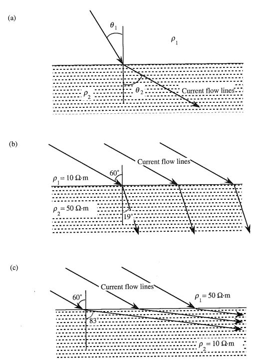
Current Flow Lines and Current Density

The preceding discussion presents us with sufficient information to make a qualitative assessment of current flow lines and, more importantly, current density distribution when a horizontal interface is present. As a first step in this process, we must investigate what happens to the orientation of flow lines and equipotentials when crossing a boundary separating regions of differing conductivities or resistivities. Hubbert (1940, p. 844–846) demonstrated that the flow lines follow a tangent relationship such that

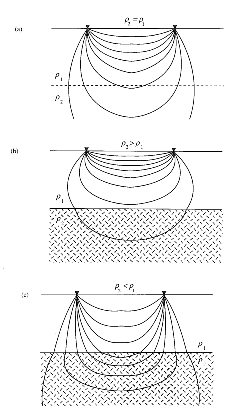
$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{\rho_2}{\rho_1} \qquad (5\text{-}19)$$

where θ and ρ are as defined in Figure 5-12(a). If the resistivity ρ_2 of the deeper material is greater, then the flow lines bend in toward the normal to the interface (Fig. 5-12(b)) and, as a consequence, are more widely spaced. However, if the reverse is true, as in Figure 5-12(c), the flow lines bend away from the normal, become oriented more parallel to the interface, and are closer together.

Refraction of Flow Lines



Refraction of Flow Lines



Apparent Resistivity

Apparent Resistivity

When we derived Eq. 5-15, we assumed a homogeneous, isotropic subsurface. As demonstrated previously, any combination of electrode spacings and current results in a potential difference that provides the correct value for the resistivity of the subsurface (as of course should be the case if our equation is correct). Once the subsurface is nonhomogeneous, the value determined for the resistivity is extremely unlikely to equal the resistivity of the material in which the electrodes are inserted. Equation 5-15 thus defines a different quantity, which is termed the *apparent resistivity* ρ_a. Inasmuch as nonhomogeneity is the rule, we write

$$\rho_a = \frac{2\pi\Delta V}{i}\left(\frac{1}{\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{r_3} + \frac{1}{r_4}}\right) \qquad (5\text{-}20)$$

The question we now face is, What does this equation tell us? How do we interpret apparent resistivity values in terms of the subsurface geology?

Apparent Resistivity

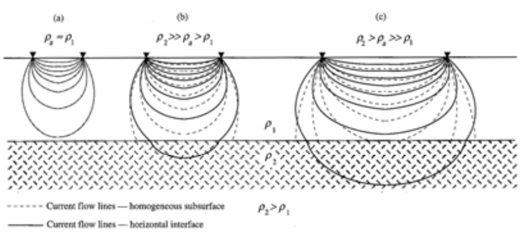


Figure 5-14 Effect on apparent resistivity as electrode spacing is increased. Dashed black lines represent current flow line distribution for homogeneous subsurface. Solid lines represent actual current flow lines due to horizontal interface. As the distance between current electrodes is increased, ρ_a approaches the value of ρ_2.

Apparent Resistivity