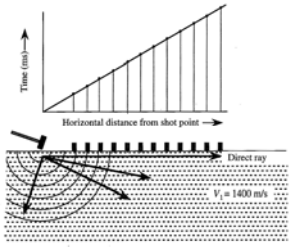
Direct Waves – Velocity from x-t



$$\text{time} = \frac{\text{path}}{\text{velocity}} \quad (3\text{-}1)$$

$$t = \frac{x}{V_1} \quad (3\text{-}2)$$

$$\frac{dt}{dx} = \frac{1}{V_1} \quad (3\text{-}3)$$

$$\text{slope} = \frac{1}{V_1} \quad (3\text{-}4)$$

Direct Wave Arrivals

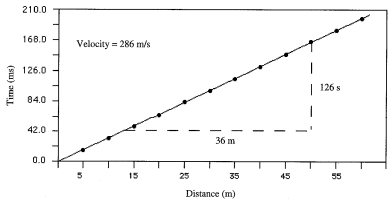


Figure 3-2 Travel-time curve (time-distance graph) illustrating only direct wave arrivals. The velocity is 286 m/s as determined from the inverse of the slope.

$$V_1 = \frac{1}{\text{slope}} \quad (3\text{-}5)$$

Head Wave Distance vs. Time

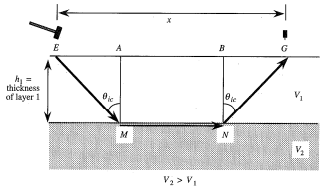


Figure 3-3 Diagram illustrating symbols used in derivation of time of travel for critically refracted ray.

The total time of travel must be

$$\text{time} = \frac{EM}{V_1} + \frac{MN}{V_2} + \frac{NG}{V_1} \quad (3\text{-}6)$$

Head Wave – Travel Time Eqn.

The total time of travel must be

$$\text{time} = \frac{EM}{V_1} + \frac{MN}{V_2} + \frac{NG}{V_1} \qquad (3\text{-}6)$$

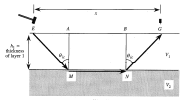


Figure 3-3 Diagram illustrating symbols used in derivation of time of travel for critically refracted ray.

Since

$$\cos\theta_{ic} = \frac{h_1}{EM} \quad \text{and} \quad EM = NG$$

then

$$EM = NG = \frac{h_1}{\cos\theta_{ic}}$$

Also,

$$EA = BG = h_1\tan\theta_{ic} \quad \text{and} \quad MN = x - 2h_1\tan\theta_{ic}$$

$$\text{time} = \frac{h_1}{V_1\cos\theta_{ic}} + \frac{x - 2h_1\tan\theta_{ic}}{V_2} + \frac{h_1}{V_1\cos\theta_{ic}} \qquad (3\text{-}8)$$

Head Wave – Travel Time Eqn.

$$\text{time} = \frac{2h_1}{V_1\cos\theta_{ic}} - \frac{2h_1\tan\theta_{ic}}{V_2} + \frac{x}{V_2} \qquad (3\text{-}9)$$

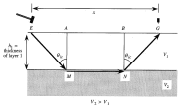


Figure 3-3 Diagram illustrating symbols used in derivation of time of travel for critically refracted ray.

Next, we utilize the relationships

$$\tan\theta_{ic} = \frac{\sin\theta_{ic}}{\cos\theta_{ic}} \quad \text{and} \quad \sin\theta_{ic} = \frac{V_1}{V_2}$$

to change Eq. 3-9 to

$$\text{time} = \frac{2h_1}{V_1\cos\theta_{ic}} - \frac{2h_1\sin^2\theta_{ic}}{V_1\cos\theta_{ic}} + \frac{x}{V_2}$$

Consolidating, we obtain

$$\text{time} = \frac{2h_1(1 - \sin^2\theta_{ic})}{V_1\cos\theta_{ic}} + \frac{x}{V_2} \qquad (3\text{-}11)$$

Using the familiar relationship

$$\sin^2\theta_{ic} + \cos^2\theta_{ic} = 1$$

and the identities noted previously, we can substitute to obtain

$$\text{time} = \frac{2h_1\cos\theta_{ic}}{V_1} + \frac{x}{V_2} \qquad (3\text{-}12)$$

$$\text{time} = \frac{2h_1(1 - (V_1^2/V_2^2))^{1/2}}{V_1} + \frac{x}{V_2} \qquad (3\text{-}13)$$

$$\text{time} = \frac{2h_1(V_2^2 - V_1^2)^{1/2}}{V_2V_1} + \frac{x}{V_2} \qquad (3\text{-}14)$$

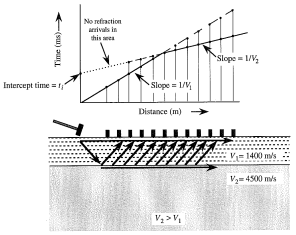


Figure 3-4 Generalized diagram illustrating ray paths in a material with one horizontal discontinuity. Time-distance relationships for both the direct and refracted rays are shown in the travel-time curve.

$$\text{time} = \frac{2h_1(V_2^2 - V_1^2)^{1/2}}{V_2V_1} + \frac{x}{V_2} \qquad (3\text{-}14)$$

With the final form of our travel-time equation (3-14) directly in front of us, let's follow the same procedure as we did for the direct wave and take the first derivative of the equation with respect to x. This provides a perhaps surprisingly simple result:

$$\frac{dt}{dx} = \frac{1}{V_2} \qquad (3\text{-}15)$$

TABLE 3-1 Arrival Times for Direct and Refracted Waves

Distance from Shot (m)	Arrival Times (ms)	
	Direct Wave	Refracted Wave
0	0.00	13.58
3	2.14	14.24
6	4.29	14.91
9	6.43	15.58
12	8.57	16.24
15	10.71	16.91
18	12.86	17.58
21	15.00	18.24
24	17.14	18.91
27	19.29	19.58
30	21.43	20.24
33	23.57	20.91
36	25.71	21.58
39	27.86	22.24
42	30.00	22.91
45	32.14	23.58
48	34.29	24.24
51	36.43	24.91
54	38.57	25.58
57	40.71	26.24
60	42.86	26.91
63	45.00	27.58
66	47.14	28.24
69	49.29	28.91

Velocity 1-P (m/s)	**1400**
Velocity 2-P (m/s)	**4500**
Depth (m)	**10**

Shot offset (m)	**0**
Geophone interval (m)	**3**

No refracted wave intervals before (ms)	6.55

Thickness from Intercept Time

Determining Thickness

Examine Figure 3-4 once again. The straight line passing through the arrival times for the critically refracted ray can be extended until it intersects the time axis. This time is termed the *intercept time* t_i. Recall that it has no real physical significance, because no refractions arrive at the energy source ($x = 0$). However, at $x = 0$ our travel-time Eq. 3-14 reduces to

$$\text{time} = t_i = 2h_i \frac{(V_2^2 - V_1^2)^{1/2}}{V_2 V_1} \tag{3-16}$$

and, therefore,

$$h_i = \frac{t_i}{2} \frac{V_2 V_1}{(V_2^2 - V_1^2)^{1/2}} \tag{3-17}$$

Thus, for a single horizontal interface, if we can determine times for direct and refracted arrivals from a field seismogram, we can calculate the thickness of the material above the interface and the velocities of the materials above and below the interface.

Thickness from Crossover Distance

At x_{co} the times of travel for the direct wave and the refracted wave are equal, and, therefore,

$$\text{time}_{\text{direct wave}} = \frac{x_{co}}{V_1} \qquad \text{(see 3-2)}$$

$$\text{time}_{\text{refracted wave}} = \frac{2h_i(V_2^2 - V_1^2)^{1/2}}{V_2 V_1} + \frac{x_{co}}{V_2} \qquad \text{(see 3-14)}$$

and

$$\frac{x_{co}}{V_1} = \frac{2h_i(V_2^2 - V_1^2)^{1/2}}{V_2 V_1} + \frac{x_{co}}{V_2} \tag{3-18}$$

If we rearrange terms, we arrive at

$$\frac{h_i(V_2^2 - V_1^2)^{1/2}}{V_2 V_1} = \frac{x_{co}}{2}\left(\frac{V_2 - V_1}{V_2 V_1}\right) \tag{3-19}$$

and

$$h_i = \frac{x_{co}}{2}\left(\frac{V_2 - V_1}{V_2 V_1}\right)\left(\frac{V_2 V_1}{(V_2^2 - V_1^2)^{1/2}}\right) \tag{3-20}$$

The $V_1 V_2$ terms cancel, and remembering that $(V_2^2 - V_1^2) = (V_2 - V_1)(V_2 + V_1)$, we can simplify Eq. 3-20 to

$$h_i = \frac{x_{co}}{2}\left(\frac{V_2 - V_1}{V_2 + V_1}\right)^{1/2} \tag{3-21}$$

It is mainly a matter of convenience whether we use Eq. 3-21 or Eq. 3-17 to determine layer thickness. However, because determination of x_{co} involves fitting *two* lines to field data, whereas t_i requires only one line, t_i usually can be determined with more accuracy.

Critical Distance (no head waves)

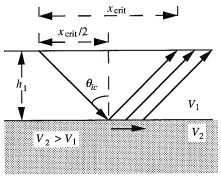


Figure 3-5 This diagram illustrates the relationships for calculating critical distance. Critical distance is the minimum distance from the energy source at which the first critical refraction can be received.

Critical Distance

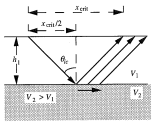


Figure 3-5 This diagram illustrates the relationships for calculating critical distance. Critical distance is the minimum distance from the energy source at which the first critical refraction can be received.

Derivation of an equation to calculate critical distance is straightforward (which you already know if you solved Problem 2-18).

$$\tan \theta_{ic} = \frac{x_{crit}/2}{h_1} \qquad (3\text{-}22)$$

and, since

$$\sin \theta_{ic} = \frac{V_1}{V_2},$$

then

$$\tan\left[\sin^{-1}\left(\frac{V_1}{V_2}\right)\right] = \frac{x_{crit}/2}{h_1} \qquad (3\text{-}23)$$

and

$$x_{crit} = 2h_1 \tan\left[\sin^{-1}\left(\frac{V_1}{V_2}\right)\right] \longrightarrow x_{crit} = \frac{2h_1}{\left[\left(V_2/V_1\right)^2 - 1\right]^{1/2}} \qquad (3\text{-}25)$$

Picking Arrivals on a Seismogram

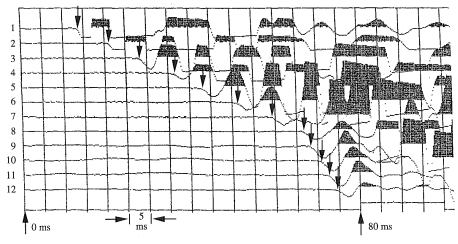


Figure 3-6 Field seismogram from the Connecticut Valley, Massachusetts. Geophone traces are labeled 1–12. The first geophone is located 5 m from the energy source. The geophone interval is 3 m. First breaks for each trace are indicated by a downward directed arrow. Timing lines are at 5-ms intervals. The record encompasses 100 ms. This seismogram exhibits a classic two-layer pattern.

Seismogram Data

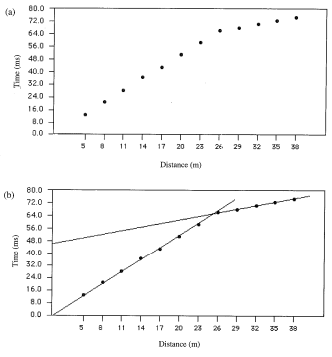


Figure 3-7 (a) Time-distance data from the seismogram in Figure 3-6. (b) Lines drawn through data points in (a).

Mohorovicic Discontinuity

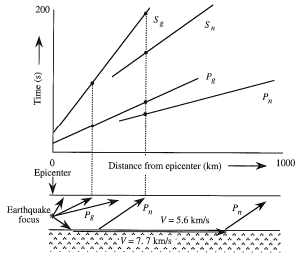


Figure 3-8 Travel-time curves simplified from Mohorovičić's original plots. P_g is the direct compressional wave, and P_n is the refracted compressional wave. Terminology for shear waves S is similar. The section below the travel-time curve illustrates Mohorovičić's interpretation of the travel-time curves.

Multiple Horizontal Layers

We know from Eq. 2-25 (Snell's law) that

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{V_1}{V_2}$$

and, since $\theta_r = \theta_c$, then

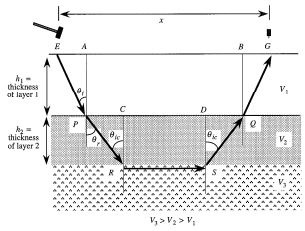


Figure 3-9 Diagram illustrating symbols used in derivation of time of travel for ray critically refracted along the second interface in a three-layer case.

2nd Interface Head Wave x-t eqn.

and, finally

$$\text{time} = \frac{x}{V_3} + \frac{2h_1(V_3^2 - V_1^2)^{1/2}}{V_3V_1} + \frac{2h_2(V_3^2 - V_2^2)^{1/2}}{V_3V_2} \qquad (3\text{-}33)$$

Once again we finish with an equation for a straight line. As you probably have observed by now, if we once again take a derivative—voila!

$$\frac{dt}{dx} = \frac{1}{V_3} \qquad (3\text{-}34)$$

$$h_2 = \left[t_{i_2} - \frac{2h_1(V_3^2 - V_1^2)^{1/2}}{V_3V_1}\right] \frac{V_2V_3}{2(V_3^2 - V_2^2)^{1/2}} \qquad (3\text{-}35)$$

2 Interface Example

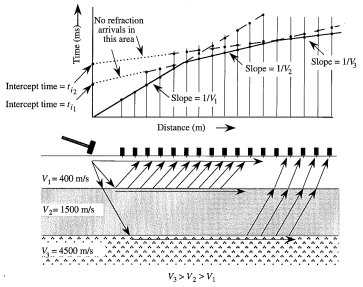


Figure 3-10 Generalized diagram illustrating ray paths in a material with two horizontal interfaces. Time-distance relationships for the direct and two critically refracted rays are shown in the travel-time curve.

2 Interface x-t Plot – computed from RefractModel

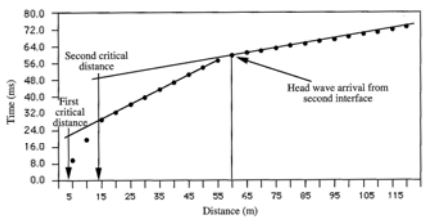


Figure 3-12 Travel-time curve based on a RefractModel plot using velocity values in Table 3-4 with $h_1 = 5$ m and $h_2 = 20$ m. Note that the first refraction from the second interface to be a first arrival is located at a distance considerably greater than the second critical distance.

Field Seismogram

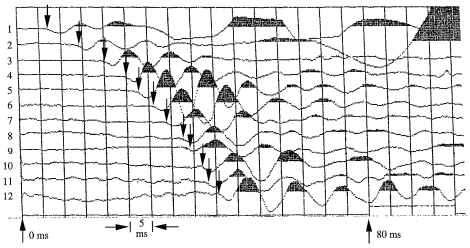


Figure 3-13 Field seismogram from the Connecticut Valley, Massachusetts. Geophone traces are labeled 1–12. The first geophone is located 3 m from the energy source. The geophone interval is 10 m. First breaks for each trace are indicated by a downward directed arrow. Timing lines are at 5-ms intervals. The record encompasses 100 ms.

2 Horizontal Interface x-t Plot – from seismogram (note: just 1 direct wave first arrival)

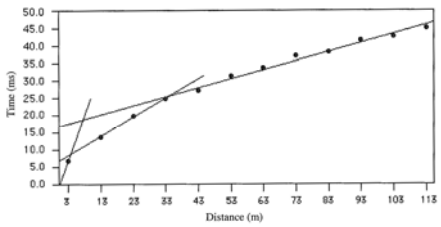


Figure 3-14 Preferred interpretation of time-distance data taken from Figure 3-13.

Horiz. Interface -> Sym. x-t plots

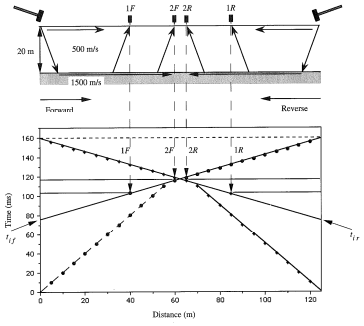


Figure 3-16 Correlation of a travel-time curve with wave paths to geophones at equal distances from an energy source for a forward and a reverse traverse.

Dipping Interface: Velocity not equal to 1/slope

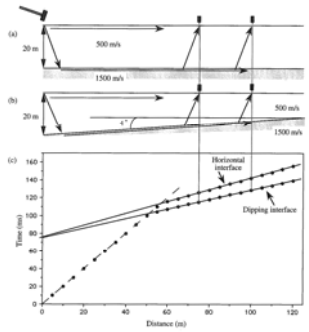


Figure 3-15 (a) Horizontal interface at a depth of 20 m with velocities above and below the interface of 500 m/s and 1500 m/s, respectively. (b) A dipping interface with identical depth to that in (a) at the site of the hammer impact and with identical velocities to (a). (c) Travel-time curves for (a) and (b).

Horizontal Interface: Symmetric x-t

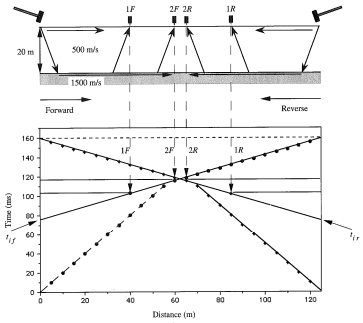


Figure 3-16 Correlation of a travel-time curve with wave paths to geophones at equal distances from an energy source for a forward and a reverse traverse.

Dipping Interface: asymmetric x-t

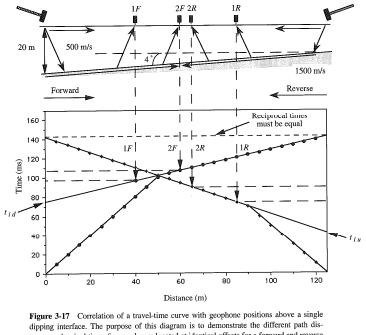


Figure 3-17 Correlation of a travel-time curve with geophone positions above a single dipping interface. The purpose of this diagram is to demonstrate the different path distances and arrival times for geophones located at identical offsets for a forward and reverse traverse.

Intercept Time gives up-dip direction

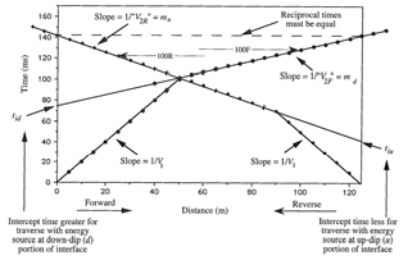


Figure 3-18 Important features of a travel-time curve for a single dipping interface. Note that reciprocal times must be equal. However, the times from forward and reverse shot points to a geophone at a given distance are not equal. This is illustrated by the arrows labeled 100F and 100R, which designate travel times to geophones located 100 m from the forward (F) and reverse (R) shots.

x-t equation for dipping interface

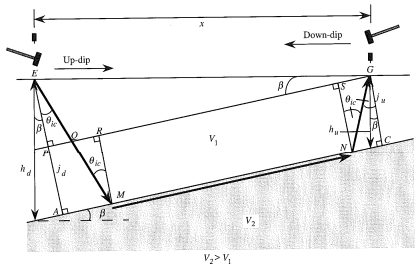


Figure 3-19 Diagram illustrating symbols used in derivation of travel-time equation for a single dipping interface.

x-t equation for dipping interface

In examining Figure 3-19, recall that θ_{ic} is measured from the perpendicular to the interface. First, we tackle the case when the energy source is located in the down-dip position (we are "shooting" up-dip).

$$\text{time}_d = \frac{EM}{V_1} + \frac{MN}{V_2} + \frac{NG}{V_1} \tag{3-40}$$

$$EM = \frac{j_d}{\cos\theta_{ic}}, \quad NG = \frac{j_u}{\cos\theta_{ic}}$$

and since

$$j_u = j_d - EP = j_d - x\sin\beta,$$

$$NG = \frac{j_d - x\sin\beta}{\cos\theta_{ic}}$$

Since $MN = PG - AM - NC$, we determine the relevant quantities

$$PG = x\cos\beta, \quad AM = j_d \tan\theta_{ic}, \quad \text{and} \quad NC = j_u \tan\theta_{ic} = (j_d - x\sin\beta)\tan\theta_{ic}$$

This gives us the basic travel-time equation

$$\text{time}_d = \frac{j_d}{V_1\cos\theta_{ic}} + \frac{x\cos\beta - j_d\tan\theta_{ic} - (j_d - x\sin\beta)\tan\theta_{ic}}{V_2} + \frac{j_d - x\sin\beta}{V_1\cos\theta_{ic}} \tag{3-41}$$

x-t equation for dipping interface

We can reduce this equation to a simpler form by using the trigonometric identities we used in our previous derivations and by remembering

$$\sin(\theta_c - \beta) = \sin\theta_c \cos\beta - \cos\theta_c \sin\beta$$
$$\sin(\theta_c + \beta) = \sin\theta_c \cos\beta + \cos\theta_c \sin\beta$$

Finally, we arrive at

$$\text{time}_d = \frac{2 j_d \cos\theta_c}{V_1} + \frac{x}{V_1}\sin(\theta_c - \beta) \quad (3\text{-}42)$$

Our next step is to move the energy source to the up-dip part of the interface to shoot down-dip. The equation we arrive at looks remarkably similar to Eq. 3-41. After simplifying once again, we have

$$\text{time}_u = \frac{2 j_u \cos\theta_c}{V_1} + \frac{x}{V_1}\sin(\theta_c + \beta) \quad (3\text{-}43)$$

These two equations are not only very similar to each other but are not as different from the horizontal single-interface case as you might have predicted. Our next step, by now almost a ritual, is to determine the slopes of the straight lines that these equations define. Taking the derivatives of Eq. 3-42 and 3-43 results in

$$\frac{d(\text{time}_d)}{dx} = \frac{\sin(\theta_c - \beta)}{V_1} \quad \text{and} \quad \frac{d(\text{time}_u)}{dx} = \frac{\sin(\theta_c + \beta)}{V_1} \quad (3\text{-}44)$$

Critical Angle and Dip of Interface from m_u and m_d

Following the terminology established in Figure 3-18 for slopes and intercept times, we begin by rewriting Eq. 3-44 in the form

$$m_d = \frac{\sin(\theta_c - \beta)}{V_1} \quad \text{and} \quad m_u = \frac{\sin(\theta_c + \beta)}{V_1} \quad (3\text{-}45)$$

Remembering that $\sin\theta_c = V_1/V_2$, we see that we can determine V_2 if we know θ_c. Of course, we also want to know the dip β of the interface. Noting the + and - signs in Eq. 3-45, we see that we can solve these slope equations for both β and θ_c. Therefore, we put these equations in the form

$$\theta_c - \beta = \sin^{-1}(V_1 m_d) \quad \text{and} \quad \theta_c + \beta = \sin^{-1}(V_1 m_u) \quad (3\text{-}46)$$

Because m_u, m_d, and V_1 are determined directly from the travel-time curve, we solve for θ_c:

$$\theta_c = \sin^{-1}(V_1 m_d) + \beta \quad \text{and} \quad \theta_c = \sin^{-1}(V_1 m_u) - \beta$$
$$2\theta_c = \sin^{-1}(V_1 m_d) + \sin^{-1}(V_1 m_u)$$

and, finally,

$$\theta_c = \frac{\sin^{-1}(V_1 m_d) + \sin^{-1}(V_1 m_u)}{2} \quad (3\text{-}47)$$

Similarly, we determine the relationship for β using

$$\beta = \theta_c - \sin^{-1}(V_1 m_d) \quad \text{and} \quad \beta = -\theta_c + \sin^{-1}(V_1 m_u)$$

to arrive at

$$2\beta = \sin^{-1}(V_1 m_u) - \sin^{-1}(V_1 m_d)$$

and

$$\beta = \frac{\sin^{-1}(V_1 m_u) - \sin^{-1}(V_1 m_d)}{2} \quad (3\text{-}48)$$

Thickness from Intercept Time

Determining Thickness

However, we are not yet quite finished. We still need to derive an equation that provides a solution for thickness, at both the up-dip and down-dip portions of the interface. At this point we know the critical angle and the velocity of the first layer, so we can express Eqs. 3-42 and 3-43 in terms of intercept times:

$$t_{id} = \frac{2 j_d \cos\theta_c}{V_1} \quad (3\text{-}49)$$

and

$$t_{iu} = \frac{2 j_u \cos\theta_c}{V} \quad (3\text{-}50)$$

Noting from Figure 3-19 that $\cos\beta = j_d/h_d$, and $\cos\beta = j_u/h_u$, we complete our task for the single dipping interface

$$j_d = \frac{t_{id} V_1}{2\cos\theta_c} \quad (3\text{-}51)$$

$$j_u = \frac{t_{iu} V_1}{2\cos\theta_c} \quad (3\text{-}52)$$

$$h_d = \frac{j_d}{\cos\beta} \quad (3\text{-}53)$$

and

$$h_u = \frac{j_u}{\cos\beta} \quad (3\text{-}54)$$