Geol 4131 — J. LORENZO

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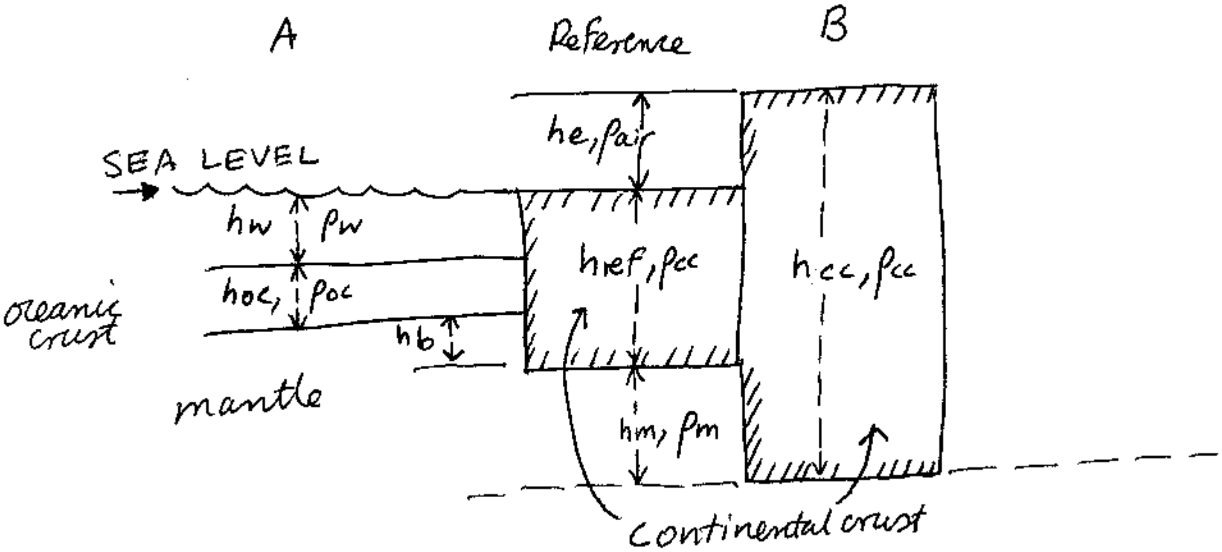
Solutions to a few simple isostatic relations

(1) height above sea level of the surface of continental crust, and the thickness of continental crust

(2) height above sealevel of the surface of continental crust and the keel height below the "sea-surface"; applicable to ice-bergs

(3) thickness of oceanic crust and water depth above oceanic crust

(4) height above sea level of the surface of oceanic crust and the thickness of oceanic crust



<u>Variables</u> :

hw – water layer thickness
hoc – oceanic crustal thickness
he – elevation
href – reference crustal thickness (30km)
hcc – continental crustal thickness
hm – mantle thickness
ρw – water density (1.0)
ρoc – oceanic crustal (2.8)
ρm – mantle density (3.3)
ρair – air density (0)
ρcc – continental crustal density (2.67)

Distance units are in km, density values are in g/cm^3.

Problem: Derive the relation between the thickness of the continental crust and the elevation above sea level of the surface of the continental crust.

Procedure: (1) Compare weights of crustal columns at B and at the reference site.

(2) Reduce the equation to a relation between two unknowns: h_e and h_{cc}

Important Relations

$$(R\text{-}1) \qquad h_{cc} = h_e + h_{ref} + h_m$$

(1)

At B At reference

$$h_{cc} \cdot \rho_{cc} = h_{ref} \cdot \rho_{cc} + h_m \cdot \rho_m$$

Substituting (R-1) for h_{cc}, we have

$$(h_e + h_{ref} + h_m)\rho_{cc} = h_{ref}\rho_{cc} + h_m\rho_m$$

$$h_e\rho_{cc} + h_m\rho_{cc} = h_m\rho_m$$

$$h_m\rho_{cc} = h_m\rho_m - h_e\rho_{cc}$$

$$h_m(\rho_{cc} - \rho_m) = -h_e\rho_{cc}$$

$$h_m \frac{(\rho_{cc} - \rho_m)}{\rho_{cc}} = -h_e$$

$$h_m \left(\frac{\rho_m - \rho_{cc}}{\rho_{cc}}\right) = h_e$$

(a) $$h_m = h_e \left(\frac{\rho_{cc}}{\rho_m - \rho_{cc}}\right)$$

Finally, by substituting (a) into (R-1) we have

$$h_{cc} = h_e + h_{ref} + h_e\left(\frac{\rho_{cc}}{\rho_m - \rho_{cc}}\right)$$

$$h_{cc} = h_e\left(1 + \frac{\rho_{cc}}{\rho_m - \rho_{cc}}\right) + h_{ref}$$

$$h_{cc} = h_e\left(\frac{\rho_m - \rho_{cc} + \rho_{cc}}{\rho_m - \rho_{cc}}\right) + h_{ref}$$

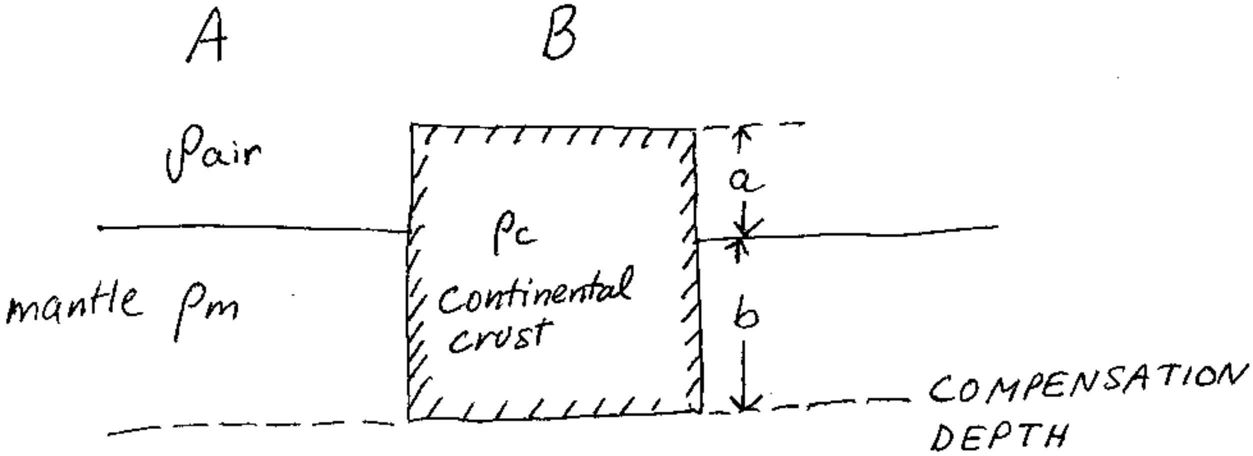
$$h_{cc} = h_e\left(\frac{\rho_m}{\rho_m - \rho_{cc}}\right) + h_{ref}$$

If we use the values of densities given on the first page, we get that

$$\frac{\rho_m}{\rho_m - \rho_{cc}} = \frac{3.3}{3.3 - 2.67} = \frac{3.3}{0.63} \doteq 5.2$$

$$h_{cc} \doteq \text{elevation}\,(5.2) + 30\text{ km}$$

e.g., elevation of 0.1 km $\Rightarrow$ $h_{cc} \doteq 30.5$ km



Variables are the same as for the other problems

<u>Procedure</u>: (1) Compare the origin of pressure at points A and B, assuming that pressures are equal.

(2) Reduce problem to the relation between two unknowns: a, or height above mantle and b, root below "sea" level

At A   At B

$$\rho_m \cdot b = \rho_c (a + b)$$

$$\rho_m \cdot b - \rho_c \cdot b = \rho_c \cdot a$$

$$b \frac{(\rho_m - \rho_c)}{\rho_c} = a$$

Substituting values as for other problems, we have

$$b = a \left(\frac{\rho_c}{\rho_m - \rho_c}\right)$$

$$= a \left(\frac{2.67}{3.3 - 2.67}\right)$$

$$b = a \cdot 4.24$$

Problem: Derive the relation between water depth and the thickness of oceanic crust.

Procedure (1) Compare pressures of two earth columns, one at A and one at the reference point.

Variables and their values are the same as for the other problems.

Useful relation (R-1) $h_b = h_{ref} - h_w - h_{oc}$

$$\underbrace{h_w \rho_w + h_{oc} \cdot \rho_{oc} + h_b \rho_m}_{(A)} = \underbrace{h_{ref} \cdot \rho_{cc}}_{(Ref)} \qquad (1)$$

$$h_{oc} \cdot \rho_{oc} = h_{ref} \rho_{cc} - h_w \rho_w - h_b \rho_m$$

Using (R-1) and substituting into the previous equation, we obtain:

$$h_{oc} \rho_{oc} = h_{ref} \rho_{cc} - h_w \rho_w - (h_{ref} - h_w - h_{oc}) \rho_m$$

$$h_{oc}(\rho_{oc} - \rho_m) = h_{ref} \rho_{cc} - h_w \rho_w - h_{ref} \rho_m + h_w \rho_m$$

$$= h_{ref}(\rho_{cc} - \rho_m) + h_w(\rho_m - \rho_w)$$

Finally:

$$h_{oc} = h_{ref} \frac{(\rho_{cc} - \rho_m)}{\rho_{oc} - \rho_m} + h_w \frac{(\rho_m - \rho_w)}{\rho_{oc} - \rho_m}$$

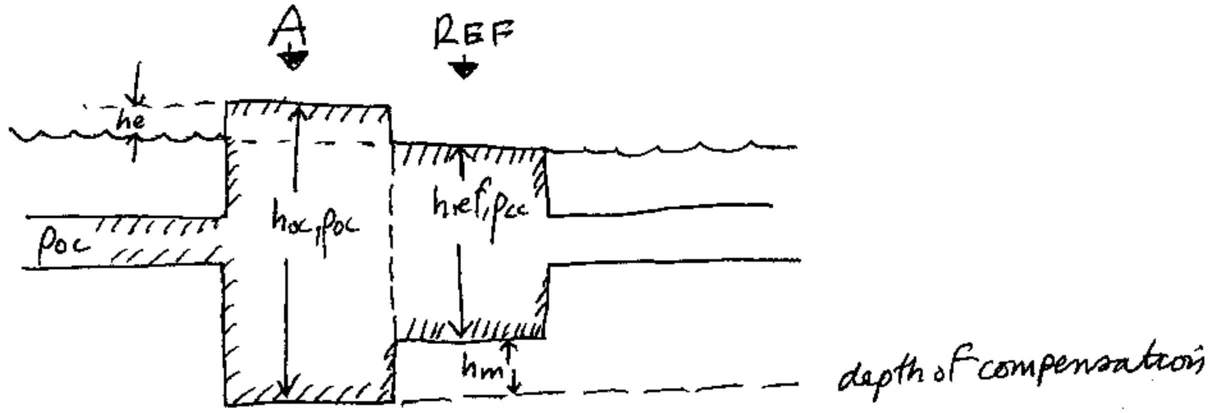
We can substitute values for the variables. Others may have also obtained the following expression:

$$h_{oc} = h_{ref}\left(\frac{\rho_m - \rho_{cc}}{\rho_m - \rho_{oc}}\right) - h_w \frac{(\rho_m - \rho_w)}{(\rho_m - \rho_{oc})}$$

$$h_{oc} = h_{ref}\left(\frac{3.3 - 2.67}{3.3 - 2.8}\right) - h_w\left(\frac{3.3 - 1}{3.3 - 2.8}\right)$$

$$h_{oc} = h_{ref}\left(\frac{0.63}{0.5}\right) - h_w\left(\frac{2.3}{0.5}\right)$$

$$h_{oc} = 37.8\,km - h_w(4.6)$$



Problem: Derive relation between elevation of oceanic crust above sea level and its thickness.

Procedure: Compare pressure at A with reference continental crust at "REF". Develop relation between hoc and he. (elevation)

Useful relation

$$h_m + h_e + h_{ref} = h_{oc} \quad (R\text{-}1)$$

At A | At "REF"

$$(a) \quad h_{oc} \cdot \rho_{oc} = h_{ref} \cdot \rho_{cc} + h_m \cdot \rho_m$$

By substituting (R-1) into (a), above we obtain

$$h_{oc} \cdot \rho_{oc} = h_{ref} \cdot \rho_{cc} + \rho_m (h_{oc} - h_e - h_{ref})$$

$$h_{oc} (\rho_{oc} - \rho_m) = h_{ref}\, \rho_{cc} + \rho_m (-h_e - h_{ref})$$

$$h_{oc}(\rho_{oc}-\rho_m) = h_{ref}(\rho_{cc}-\rho_m) - \rho_m \cdot h_e$$

$$h_{oc} = h_{ref}\frac{(\rho_{cc}-\rho_m)}{(\rho_{oc}-\rho_m)} - \frac{\rho_m \cdot h_e}{(\rho_{oc}-\rho_m)}$$

$$= h_{ref}\frac{(\rho_m-\rho_{cc})}{(\rho_m-\rho_{oc})} + h_e\frac{\rho_m}{(\rho_m-\rho_{oc})}$$

If we substitute the common values for variables that we used in other problems we obtain:

$$h_{oc} = 30\left(\frac{3.3-2.67}{0.5}\right) + h_e\left(\frac{3.3}{0.5}\right)$$

$$\underline{\underline{h_{oc} = 37.8\,km + h_e(6.6)}}$$